Multipole Hair of Higher Dimensional Schwarzschild Black Holes

Based on arXiv:1907.07622 [gr-qc] (Journal of Mathematical Physics)

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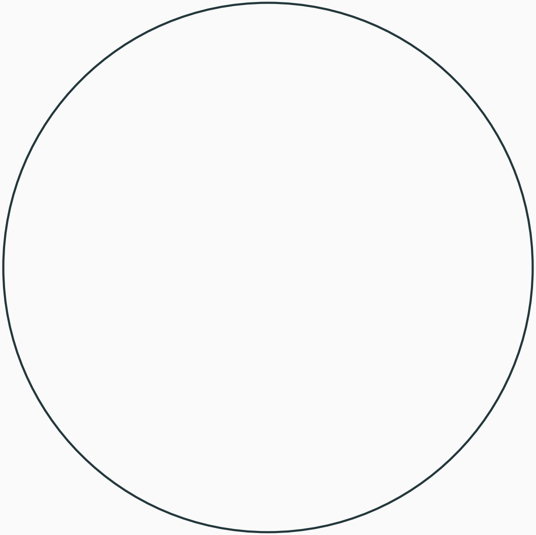
APS Far West Section, Stanford University
2 November 2019

To Be or Not to Be (a Black Hole)



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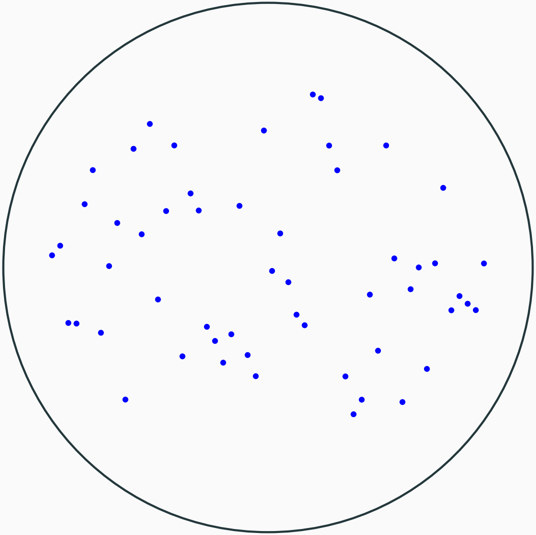

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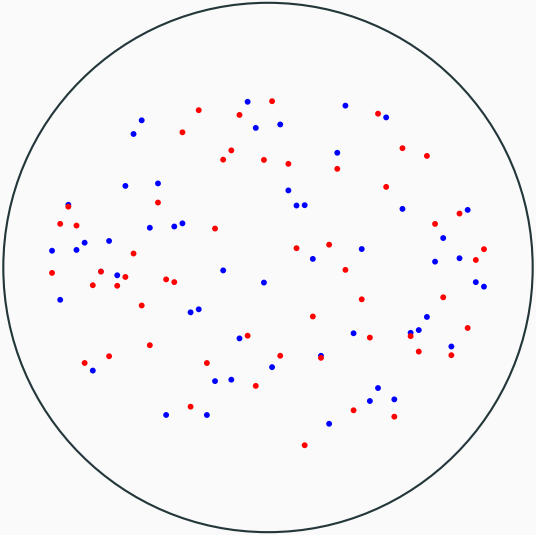

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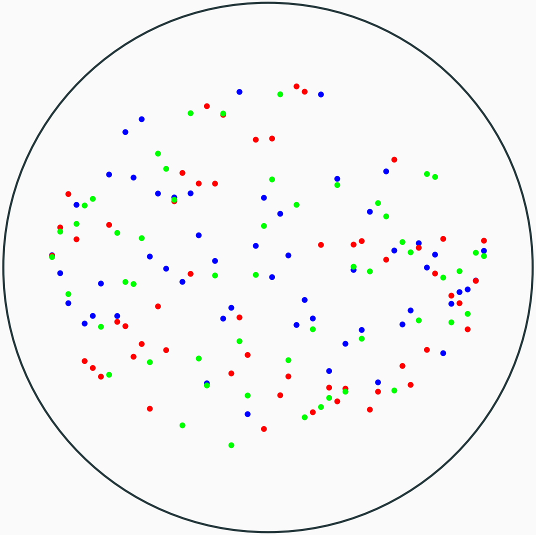

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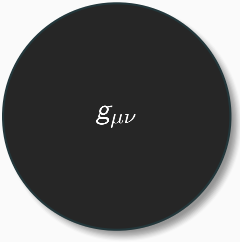

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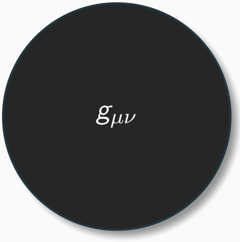

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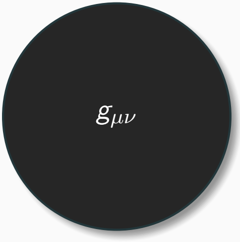

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No hair!

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Of course,

$$\partial \mathcal{B}_4 \cong S^2 \not\Longrightarrow \partial \mathcal{B}_d \cong S^{d-2}$$

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Theorem (Galloway *et al.*, 2006). Assuming dominant energy condition,

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Example (Black Saturn). 5D black hole $\mathcal{B}_5$ with horizon topology

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not precluded.

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So, higher dimensional black holes not necessarily unique.

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Uniqueness Theorems for $\partial\mathcal{B}_d \cong S^{d-2}$

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$$g_{\mathrm{ST}}(\mathrm{d}\boldsymbol{x}, \mathrm{d}\boldsymbol{x}) = -\left(1 - \frac{M}{r^{d-3}}\right)\mathrm{d}t^2 + \left(1 - \frac{M}{r^{d-3}}\right)^{-1}\mathrm{d}r^2 + r^2\underbrace{\gamma(\mathrm{d}\boldsymbol{\varphi}, \mathrm{d}\boldsymbol{\varphi})}_{S^{d-2}\text{ metric}}$$

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Theorem (Hwang, 1998). All topologically-spherical, static, asymptotically-flat, and non-degenerate vacuum solutions to Einstein equations have ST geometry, g_{ST}

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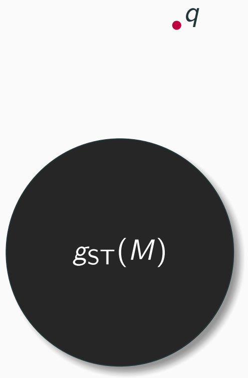

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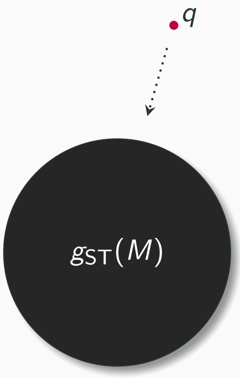

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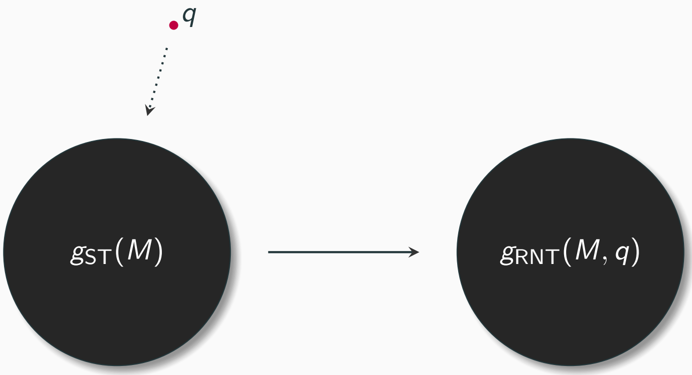

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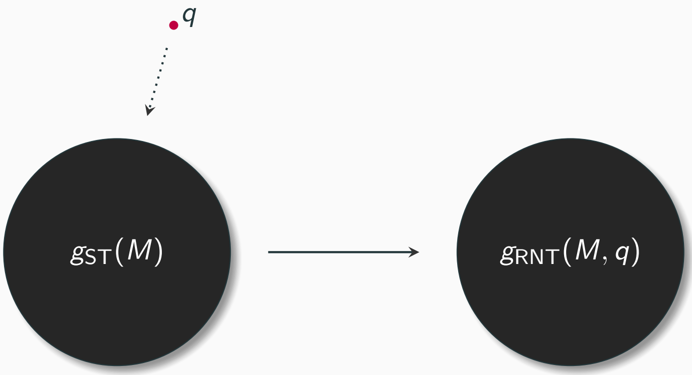

Well, not exactly...

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Electric Charge Disrupts Horizon Topology

$$g(M, q) \neq g_{RNT}(M, q)$$

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Thus,

final state is not topologically spherical

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Acknowledgements

- Brian Shuve (Harvey Mudd College)
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- Thomas Moore (Pomona College)