Punching Holes in Higher Dimensional Schwarzschild Black Holes

Based on arXiv:1907.07622 [gr-qc] (Journal of Mathematical Physics)

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2 December 2019

$$n = 3$$

Physics. — *"In what way does it become manifest in the fundamental laws of physics that space has three dimensions?"* By Prof. Dr. P. EHRENFEST. (Communicated by Prof. Dr. H. A. LORENTZ).

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$$\left.\begin{aligned} F^{(n)}_{\text{gravity}} &\propto \frac{1}{r^{n-1}} \\[2em] F^{(n)}_{\text{electric}} &\propto \frac{1}{r^{n-1}} \end{aligned}\right\} \qquad n = 3$$

A Problem in $n = 3$ Dimensions

Schwarzschild, Reissner, and Nordström

Schwarzschild Black Hole

Characteristic Properties

- electrically neutral

$$g_{\mathrm{S}}(M) \to -\left(1 - \frac{M}{r}\right) \mathrm{d}t^2 + \left(1 - \frac{M}{r}\right)^{-1} \mathrm{d}r^2 + r^2 \mathrm{d}\Omega_2^2$$

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Reissner-Nordström Black Hole

Characteristic Properties

- electrically charged

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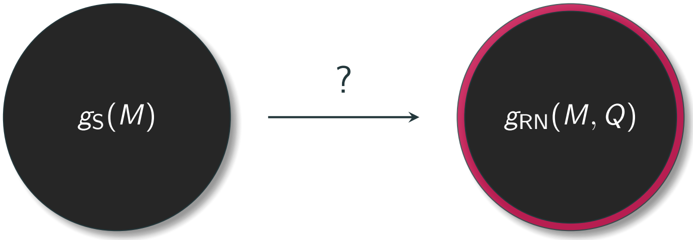
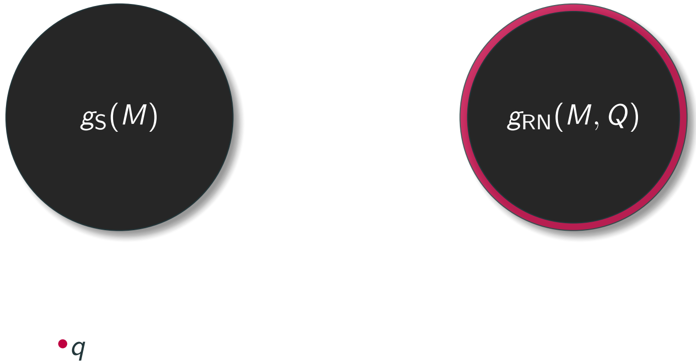
The Problem

Schwarzschild **Reissner-Nordström**



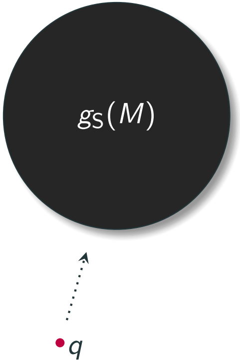
A Solution?

Schwarzschild **Reissner-Nordström**



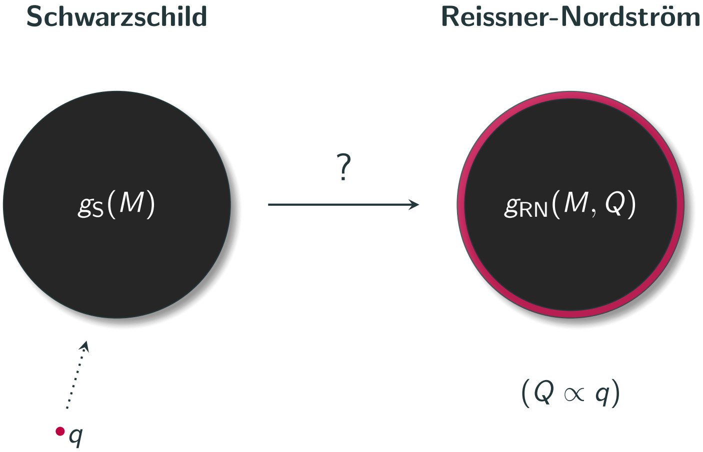
A Solution?

Schwarzschild



Reissner-Nordström

A Solution?

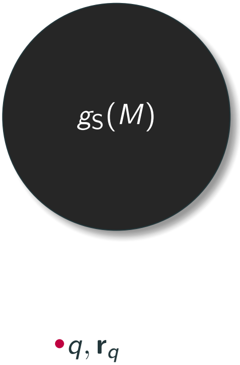


JOURNAL OF MATHEMATICAL PHYSICS VOLUME 12, NUMBER 9 SEPTEMBER 1971

Point Charge in the Vicinity of a Schwarzschild Black Hole*

Jeffrey M. Cohen
Institute for Advanced Study, Princeton, New Jersey 08540
and
Robert M. Wald†
Joseph Henry Physical Laboratory, Princeton University, Princeton, New Jersey 08540
(Received 1 February 1971)

The electrostatic field of a point charge at rest in Schwarzschild space is derived. The solution is used to study the problem of a point charge slowly lowered into a nonrotating black hole. We find that the electric field of the charge remains well behaved as the charge is lowered and that all the multipole moments except the monopole fade away. We conclude that a Reissner–Nordstrom black hole is produced.



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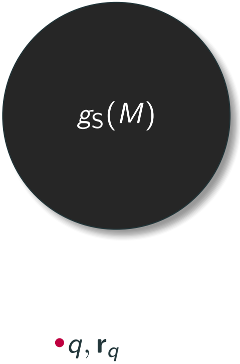

$$\Phi(\mathbf{r}, \mathbf{r}_q) \propto \frac{q}{r} + (r_q - r_s) \times \underbrace{\mathcal{O}(r^{-2})}_{\text{multipoles}}$$

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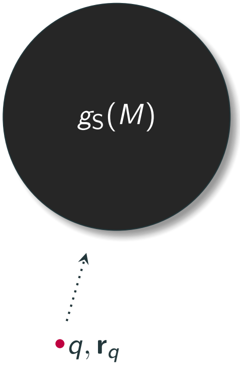

$$\Phi(\mathbf{r}, \mathbf{r}_q) \propto \frac{q}{r} + (r_q - r_s) \times \underbrace{\mathcal{O}(r^{-2})}_{\text{multipoles}}$$

$$\lim_{r_q \to r_s^+} \Phi(\mathbf{r}, \mathbf{r}_q) \propto \frac{q}{r}$$

(spherical symmetry!)

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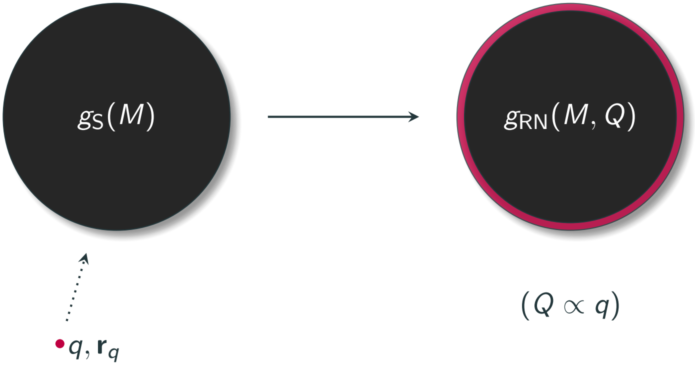
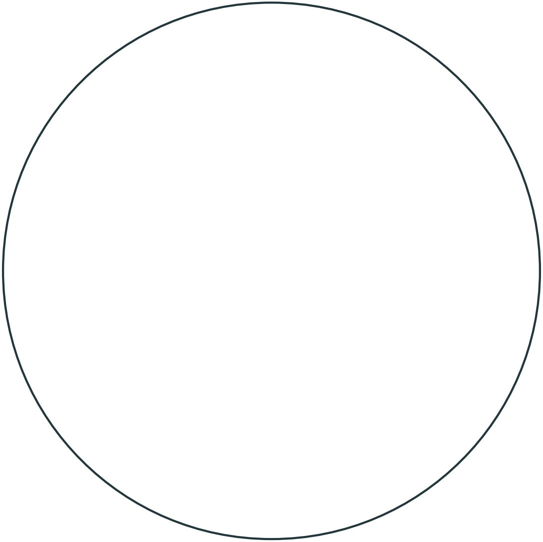
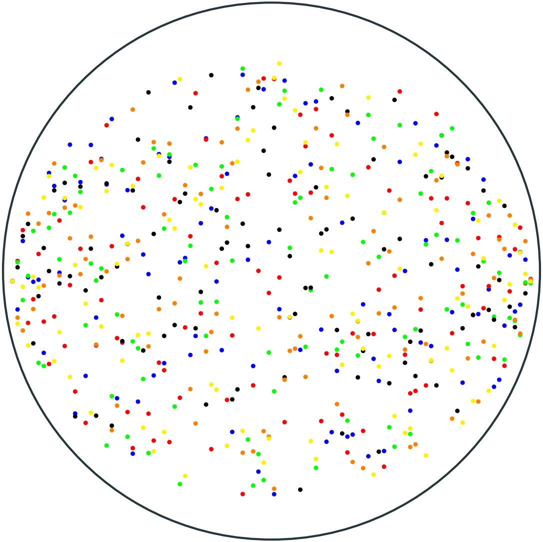


Black Hole Uniqueness, $\mathbf{n = 3}$

The No-Hair Theorem



The No-Hair Theorem



The No-Hair Theorem

W. Israel, *Phys. Rev.* **164**, 1776 (1967)

B. Carter, *Phys. Rev. Lett.* **26**, 331 (1971)

D. C. Robinson, *Phys. Rev. Lett.* **34**, 905 (1975)

The No-Hair Theorem

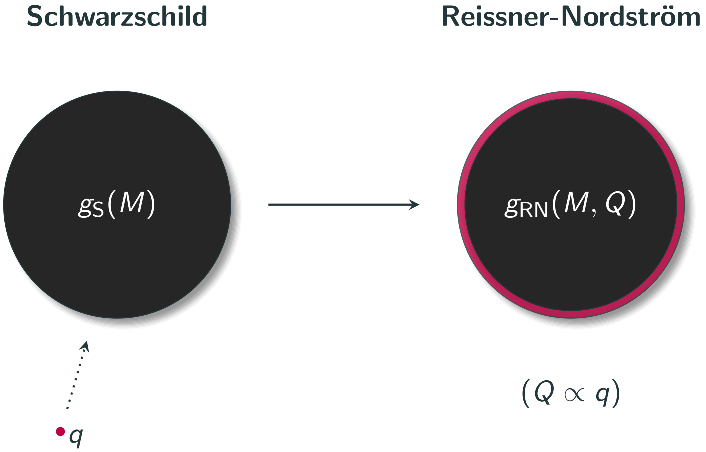
$$g = g(M, Q, J), \textbf{ not } g(\text{red}, \text{blue}, \text{green}, \ldots)$$

W. Israel, *Phys. Rev.* **164**, 1776 (1967)

B. Carter, *Phys. Rev. Lett.* **26**, 331 (1971)

D. C. Robinson, *Phys. Rev. Lett.* **34**, 905 (1975)

The "Sledgehammer" Solution

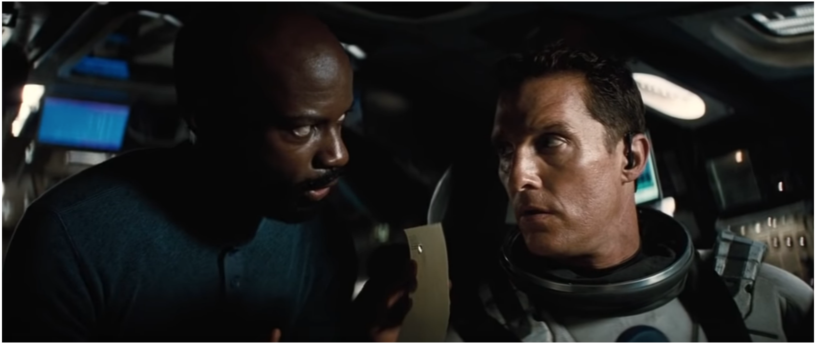


Hawking's Topology Theorem ($n = 3$)

Interstellar

Romilly: "What's a circle in three dimensions?"

Cooper: "A sphere."



https://www.youtube.com/watch?v=MywwqjqKF6Y

Hawking's Topology Theorem ($\mathbf{n = 3}$)

Theorem (Hawking, 1972). Black hole boundary $\mathcal{B}$ satisfies

$$\int_{\mathcal{B}} R \; \mathrm{d}A > 0$$

S. W. Hawking, *Commun. Math. Phys.* **25**, 152 (1972)

Hawking's Topology Theorem ($n = 3$)

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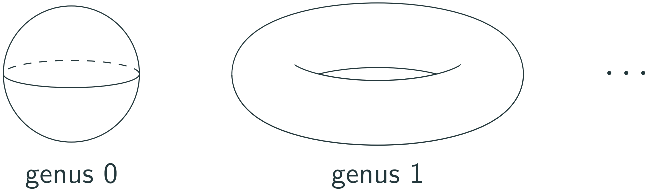
Theorem (Gauss-Bonnet). A closed 2D Riemannian manifold $\mathcal{B}$ with scalar curvature R satisfies

$$\int_{\mathcal{B}} R \, \mathrm{d}A = \pi(2 - 2 \times \text{genus})$$



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Black Hole Uniqueness, n > 3

Black Hole Topology when $\mathbf{n} > 3$

Let g be a Riemannian metric on $\mathcal{B}$ and define **conformal class**

$$[g] \equiv \left\{ h = e^{2f} g \mid f : \mathcal{B} \to \mathbf{R} \right\}$$

J. M. Lee and T. H. Parker, *Bull. Am. Math. Soc* **17**, 37 (1987)

Black Hole Topology when n > 3

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To each conformal class $[g]$, associate a **Yamabe constant**

$$\lambda([g]) = \inf_{h \in [g]} \left(\int_{\mathcal{B}} R_h \, \mathrm{d}V_h \right) \left(\int_{\mathcal{B}} \mathrm{d}V_h \right)^{-\frac{n-3}{n-1}}$$

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The greatest Yamabe constant associated with $\mathcal{B}$ is then

$$\sigma(\mathcal{B}) = \sup_{[g]} \lambda([g])$$

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Theorem (Lee and Parker, 1987). For compact, Riemannian $\mathcal{B}$, $\sigma(\mathcal{B}) > 0$ if and only if $\mathcal{B}$ carries metric of positive scalar curvature

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Black Hole Topology when n > 3

Theorem (Galloway *et al.*, 2006). Black hole boundary $\mathcal{B}$ satisfies

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I.e., $\mathcal{B}$ carries metric of positive scalar curvature

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G. J. Galloway and R. Schoen, *Commun. Math. Phys.* **266**, 571 (2006)

R. Emparan and H. S. Reall, *Phys. Rev. Lett.* **88**, 101101 (2002)

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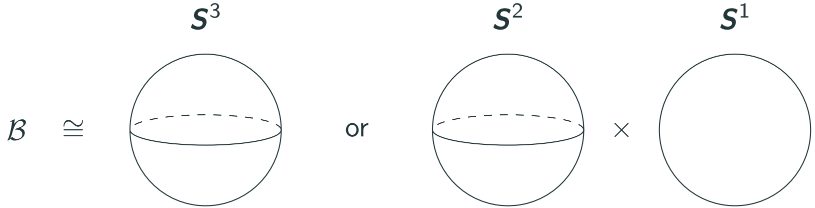
Example (Emparan *et al.*, 2002). If $n = 4$,



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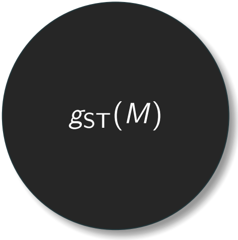
Uniqueness Theorems when $\mathcal{B} \cong S^{n-1}$

Schwarzschild-Tangherlini (ST) | **Characteristic Properties**



$$g_{\mathrm{ST}}(M) \to -\left(1-\frac{M}{r^{n-2}}\right)\mathrm{d}t^2 + \left(1-\frac{M}{r^{n-2}}\right)^{-1}\mathrm{d}r^2 + r^2\mathrm{d}\Omega_{n-1}^2$$

S. Hwang, *Geometriae Dedicata* **71**, 5 (1998)

M. Rogatko, *Phys. Rev. D* **67**, 084025 (2003); *Phys. Rev. D* **73**, 124027 (2006)

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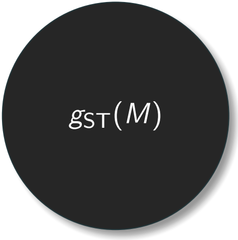

Characteristic Properties

- electrically neutral
- static
- asymptotically flat

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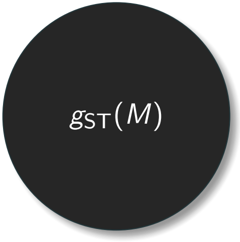

Characteristic Properties

- electrically neutral
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- asymptotically flat
- non-degenerate horizon

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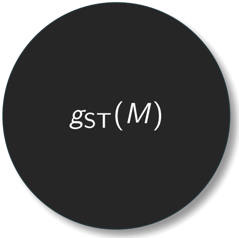

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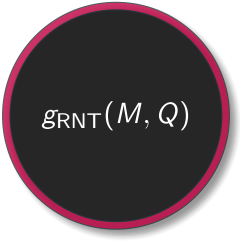
M. Rogatko, *Phys. Rev. D* **67**, 084025 (2003); *Phys. Rev. D* **73**, 124027 (2006)

Uniqueness Theorems when $\mathcal{B} \cong S^{n-1}$

Reissner-Nordström-Tangherlini (RNT)



Characteristic Properties

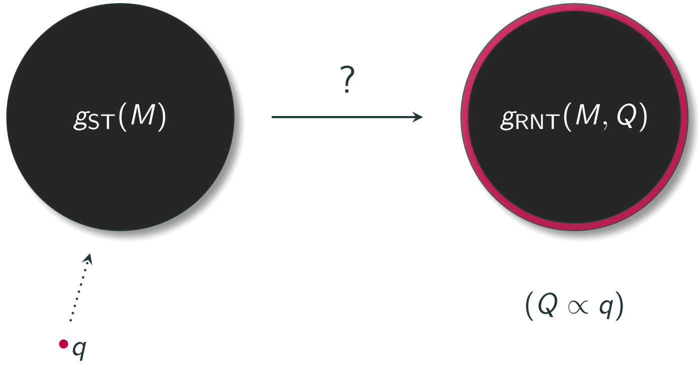
- electrically charged
- static
- asymptotically flat
- non-degenerate horizon
- $\mathcal{B} \cong \boldsymbol{S}^{n-1}$

$$g_{\mathrm{RNT}}(M, Q) \to -\left(1 - \frac{M}{r^{n-2}} + \frac{Q}{r^{2n-4}}\right) \mathrm{d}t^2 + \left(1 - \frac{M}{r^{n-2}} + \frac{Q}{r^{2n-4}}\right)^{-1} \mathrm{d}r^2 + r^2 \mathrm{d}\Omega^2_{n-1}$$

G. W. Gibbons, D. Ida, and T. Shiromizu, *Phys. Rev. D* **66**, 044010 (2002)

Same Problem, but n $>$ 3

Schwarzschild-Tangherlini **Reissner-Nordström-Tangherlini**



Charging the ST Black Hole

The ST Chart and Hyperspherical Coordinates

Schwarzschild Chart

$$(t, r, \theta, \phi) : \mathcal{M}^4 \longrightarrow \mathbf{R}^4$$

The ST Chart and Hyperspherical Coordinates

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$$(t, r, \theta, \phi) : \mathcal{M}^4 \longrightarrow \mathbf{R}^4$$

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Schwarzschild-Tangherlini Chart

$$(t, r, \varphi_1, \ldots, \varphi_{n-1}) : \mathcal{M}^{n+1} \longrightarrow \mathbf{R}^{n+1}$$

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$$\mathbf{r} \equiv (r, \boldsymbol{\varphi}) \equiv (r, \varphi_1, \ldots, \varphi_{n-1})$$

Electrostatics in ST Geometry

Task

Solve Maxwell's equations in ST spacetime $(\mathcal{M}^{n+1}, g_{\mathsf{ST}})$,

$$j^\nu = \frac{1}{\sqrt{|\det g_{\mathsf{ST}}|}}\partial_\mu \left(\sqrt{|\det g_{\mathsf{ST}}|}F^{\mu\nu}\right)$$

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Approach

Assuming electrostatic conditions with charge q at position $\mathbf{r}_q$,

$$\Delta_{\mathcal{M}^{n+1}} \Phi(\mathbf{r}, \mathbf{r}_q) = q g_{00} \delta(\mathbf{r} - \mathbf{r}_q) + \frac{r_s^{n-2}(n-2)}{r^{n-1}} \partial_r \Phi(\mathbf{r}, \mathbf{r}_q)$$

Poisson's equation!

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Approach

Assuming electrostatic conditions with charge q at position $\mathbf{r}_q$,

$$\Delta_{\mathcal{M}^{n+1}} \Phi(\mathbf{r}, \mathbf{r}_q) = q g_{00} \delta(\mathbf{r} - \mathbf{r}_q) + \frac{r_s^{n-2}(n-2)}{r^{n-1}} \partial_r \Phi(\mathbf{r}, \mathbf{r}_q)$$

Poisson's equation! Use canonical ansatz

$$\Phi(\mathbf{r}, \mathbf{r}_q) = \sum_{k \geq 0} R_k(r, r_q) Y_k(\varphi)$$

Angular Equation

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Angular Equation

$$\Delta_{\boldsymbol{S}^{n-1}} Y_k(\varphi) = -k(k+n-2)Y_k(\varphi)$$

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Hyperspherical harmonic order,

$$\Gamma_k = \frac{(2k+n-2)(n+k-3)!}{k!(n-2)!}$$

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Degree and order parameters lead to completeness relation

$$\sum_{k \geq 0} \sum_{l=1}^{\Gamma_k} \hat{Y}_k^l(\varphi_1) Y_k^l(\varphi_2) = \delta(\varphi_1 - \varphi_2)$$

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Radial Equation

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Electrostatics in ST Geometry

Radial Equation

$$\frac{d}{dr}\left[r^{n-1}g_{00}R_k'\right]-r^{n-3}k(k+n-2)R_k-(n-2)r_s^{n-2}R_k'=qg_{00}r^{n-1}\delta(r-r_q)$$

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For $r \neq r_q$, solutions are hypergeometric series

$$R_k^{(\alpha)}(r,r_s)=\sum_{m\geq 0}\alpha_{k,m}\rho^{m+\chi_k^+} \quad \text{and} \quad R_k^{(\bar{\alpha})}(r,r_s)=\sum_{m\geq 0}\bar{\alpha}_{k,m}\rho^{m-\chi_k^-}$$

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For $r = r_q$, integrate radial equation

$$\lim_{\epsilon\to 0^+}\int_{r_q-\epsilon}^{r_q+\epsilon}\mathrm{LHS}\,\mathrm{d}r = qg_{00}(r_q)r_q^{n-1}$$

treating R_k as a linear combination of $R_k^{(\alpha)}$ and $R_k^{(\bar{\alpha})}$

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Electrostatics in ST Geometry

Radial Equation

For $r > r_q$, ultimately obtain

$$R_k(r, r_q) \propto R_k^{(\alpha)}(r) R_k^{(\bar{\alpha})}(r_q)$$

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Electrostatics in ST Geometry

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Upshot,

$$\lim_{r_q \to r_s^+} R_k(r, r_q) \propto \begin{cases} 1 & \text{if } k = 0 \\ 0 & \text{if } k \in \mathbf{Z}^+ \text{ and } (n-2) \,|\, k \\ \frac{\Gamma\left(-\frac{2k}{n-2}\right)}{\Gamma\left(-\frac{k}{n-2}\right)\Gamma\left(1-\frac{k}{n-2}\right)} & \text{otherwise} \end{cases}$$

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There exist multipole moments ("multipole hair") when $n > 3$

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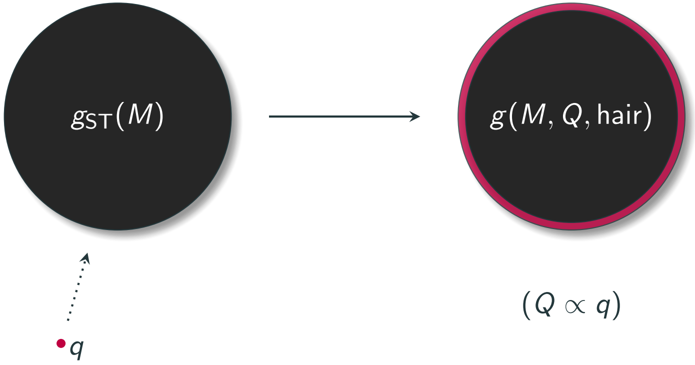
Multipole Hair of ST Black Holes

Schwarzschild-Tangherlini

Something Else
~~Reissner-Nordström-Tangherlini~~



$(Q \propto q)$

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Charge-Induced Topology Change

ST + Charge Black Hole

Characteristic Properties

- electrically charged

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Charge-Induced Topology Change

ST + Charge Black Hole

Characteristic Properties

- electrically charged
- static
- asymptotically flat
- non-degenerate horizon (likely)
- $\mathcal{B} \not\cong \boldsymbol{S}^{n-1}$

Final state is **not** topologically spherical

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Conclusion

Main Takeaways

1. Higher dimensional black holes less constrained than 3D ones

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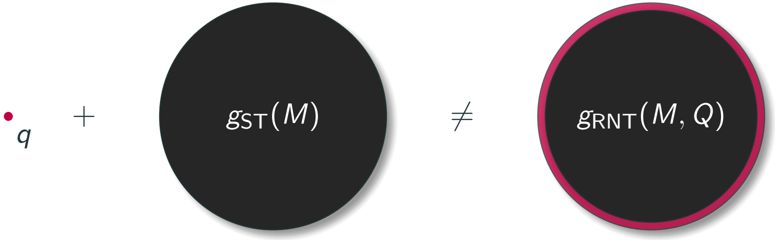
2.



Something odd (a topology change) happens as charge falls in

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- Jason Gallicchio (Harvey Mudd College)
- Brian Shuve (Harvey Mudd College)
- Thomas Moore (Pomona College)

Radial Equation

$$R_k'' + \frac{n-1}{r} R_k' - \frac{k(k+n-2) r^{n-4}}{r^{n-2} - r_s^{n-2}} R_k = 0$$

Define $\rho = \left(\frac{r_s}{r}\right)^{n-2}$,

$$\rho^2(\rho - 1)\ddot{R}_k + \frac{k(k+n-2)}{(n-2)^2} R_k = 0$$

Hypergeometric function! Solve around $\rho = 0$ so $r \in (r_s, +\infty)$:

$$R_k^{(\alpha)}(r, r_s) = \sum_{m \geq 0} \alpha_{k,m} \rho^{m + \chi_k^+}$$
$$R_k^{(\bar{\alpha})}(r, r_s) = \sum_{m \geq 0} \bar{\alpha}_{k,m} \rho^{m - \chi_k^-}$$

Radial Equation

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Here, χ_k^+ and χ_k^- are roots of indicial equation for radial equation,

$$\chi_k^{\pm} = \frac{1 \pm 1}{2} + \frac{k}{n-2}$$

Then,

$$\tilde{\alpha}_{k,m} \equiv \frac{\alpha_{k,m}}{\alpha_{k,0}} = \frac{(\chi_k^+)_m(\chi_k^-)_m}{m!(2\chi_k^+)_m}$$

and

$$\tilde{\bar{\alpha}}_{k,m} \equiv \frac{\bar{\alpha}_{k,m}}{\bar{\alpha}_{k,0}} = \frac{(-\chi_k^-)_m(-\chi_k^+)_m}{m!(-2\chi_k^-)_m}$$

Radial Equation

Matching asymptotic part with electrostatics in $n+1$ Minkowski,

$$R_k^{(\alpha)}(r, r_s) = r_s^{-(k+n-2)} \sum_{m \geq 0} \tilde{\alpha}_{k,m} \left(\frac{r_s}{r}\right)^{k+(m+1)(n-2)}$$

$$R_k^{(\bar{\alpha})}(r, r_s) = r_s^{k} \sum_{m \geq 0} \tilde{\bar{\alpha}}_{k,m} \left(\frac{r}{r_s}\right)^{k-m(n-2)}$$

Green's Function

$$\Phi(\mathbf{r}, \mathbf{r}_q) \propto q \sum_{k \in \Lambda} \sum_{l=1}^{\Gamma_k} \sum_{m \geq 0} \frac{\tilde{\alpha}_{k,m} R_k^{(\bar{\alpha})} \hat{Y}_k^l(\varphi_q) Y_k^l(\varphi)}{r_s^{k+n-2}(2k+n-2)} \left(\frac{r_s}{r}\right)^{k+(m+1)(n-2)}$$

Here:

$$\Lambda = \{k \in \mathbf{Z}^+ : (n-2) \nmid k\} \cup \{0\}$$

$$\Gamma_k = \frac{(2k+n-2)(n+k-3)!}{k!(n-2)!}$$

For each $k \in \Lambda$, there exists a μ_k-pole moment provided

$$\mu_k \equiv k \pmod{n-2}$$