Quantum Computing Near Deutschian Closed Timelike Curves

Matthew Fox, Nick Koskelo, and David Sobek

2 May 2019

Harvey Mudd College

Roadmap

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- Closed Timelike Curves (CTCs)

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  $\rightarrow$ Consistent with general relativity?

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  - $\rightarrow$ Clone quantum states, distinguish non-orthogonal states, etc.
  - $\rightarrow$ Classical and quantum computers simulable in **PSPACE**

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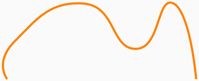

$\mathcal{M}$

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What about **timelike**?

Closed Timelike Curves (CTCs)

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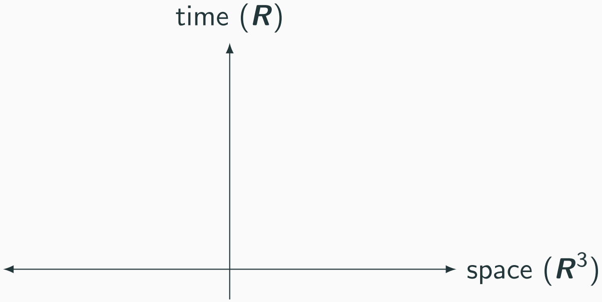

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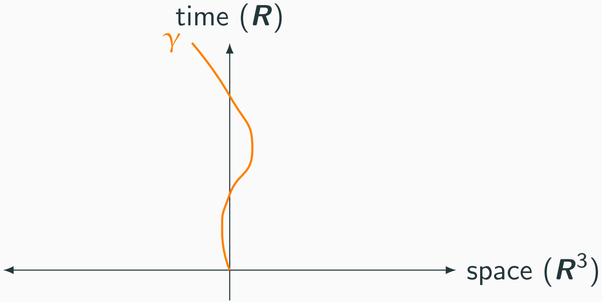

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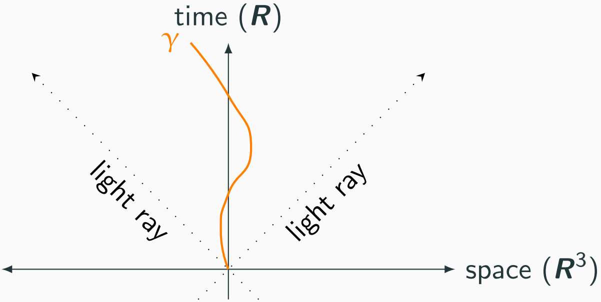

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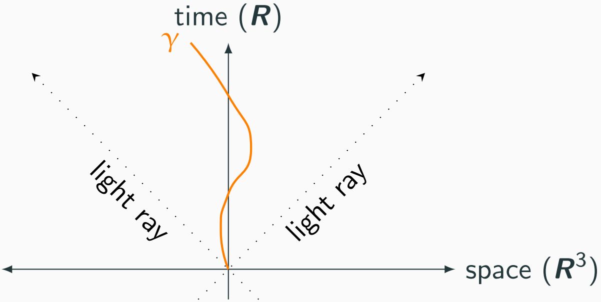

The curve γ is **timelike** if every tangent vector to γ points *within* the direction of the light ray lines.

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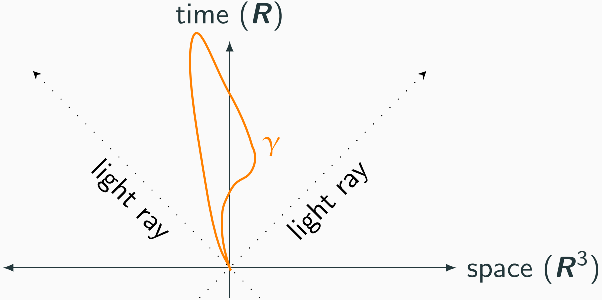

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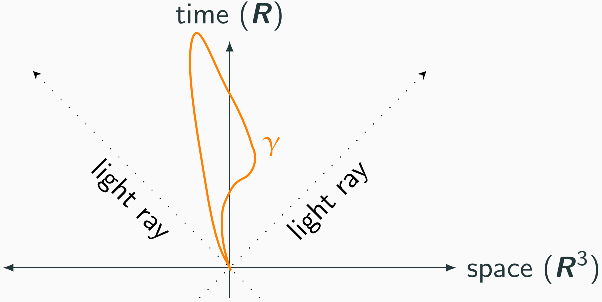

Well, not quite... flat space-time geometry forbids CTCs.

General Relativistic Considerations

Key idea: Matter warps space-time geometry away from "flat"

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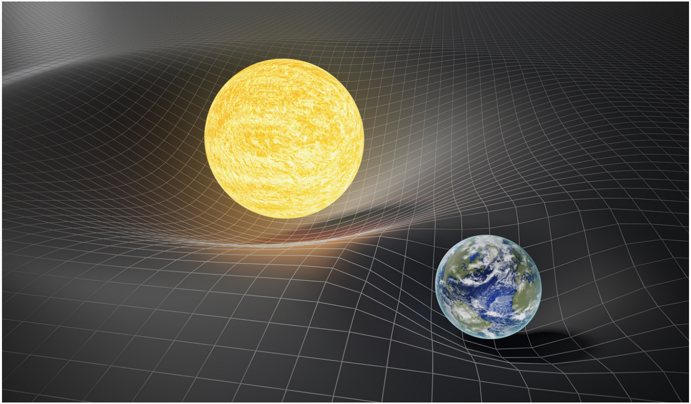

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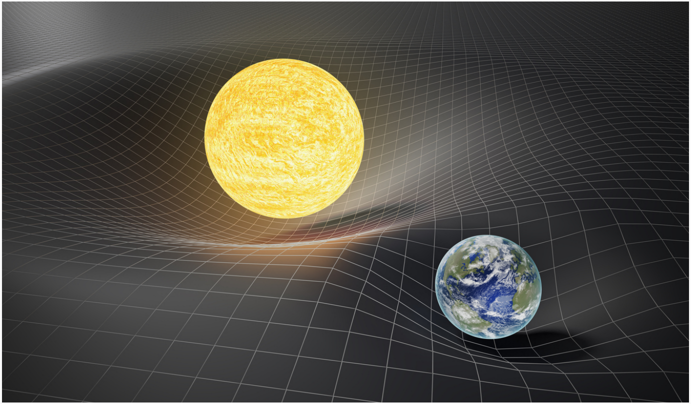

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Judicious matter distribution $\implies$ CTC-admitting space-time?

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There exist many CTC-admitting space-times consistent with GR:

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Question: What are the computational implications of CTCs?

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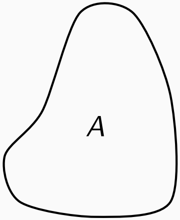
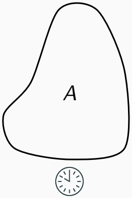


General CTC Mechanics

Absent interactions, system A remains unchanged as time evolves



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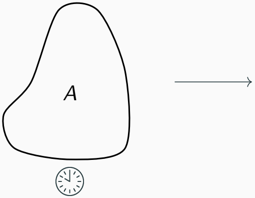

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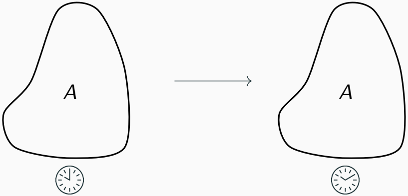

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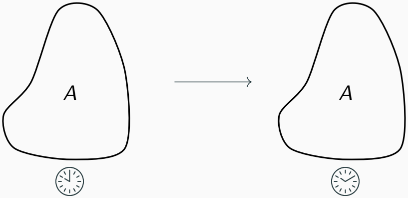

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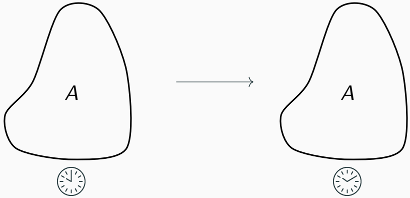

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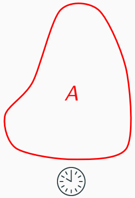

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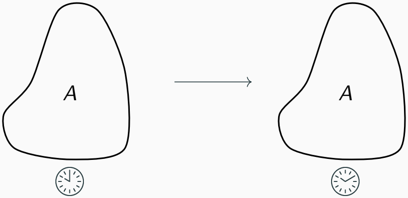

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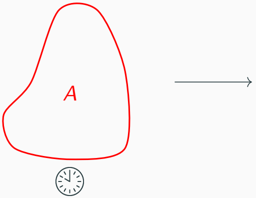

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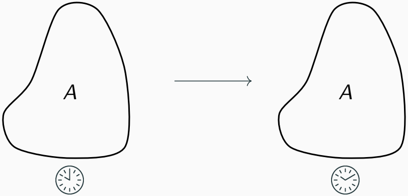

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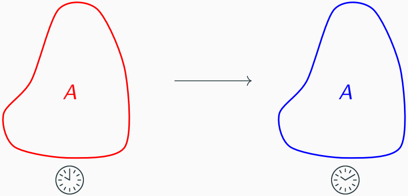
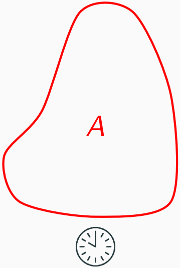


General CTC Mechanics

younger

older



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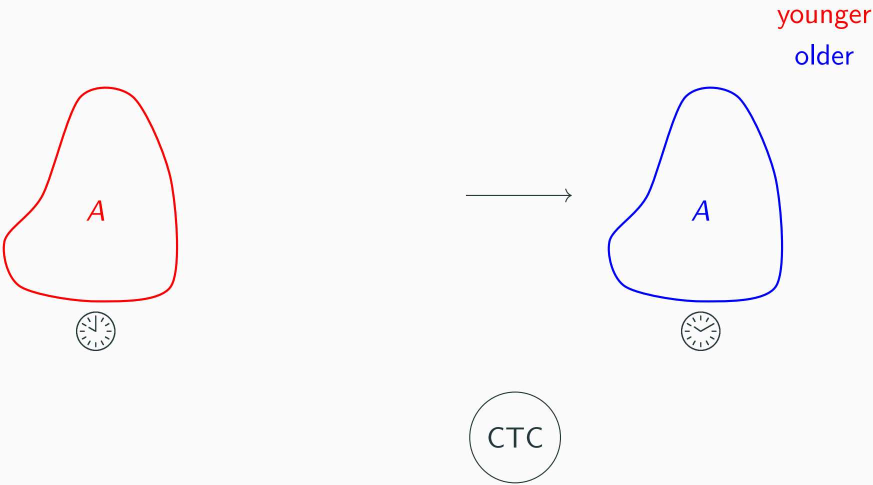

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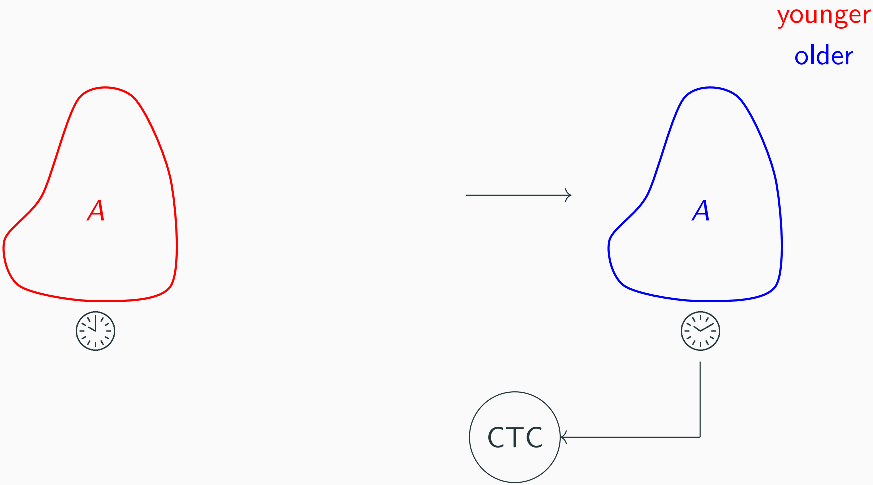

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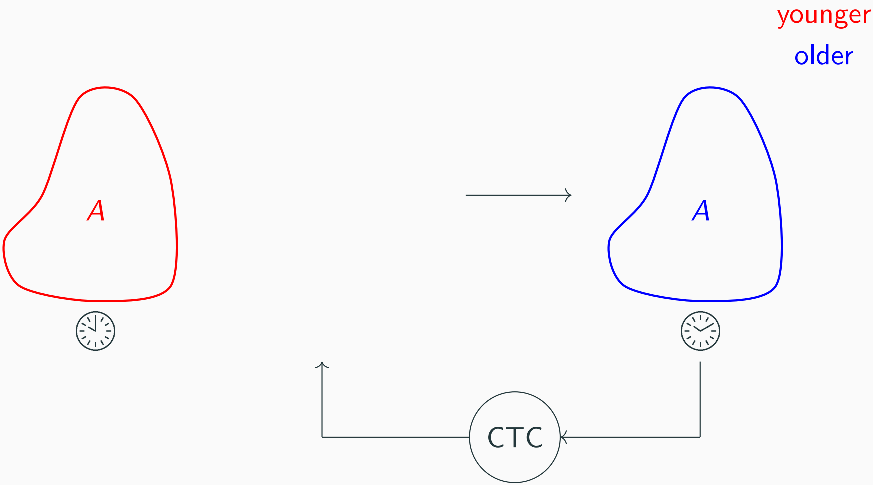

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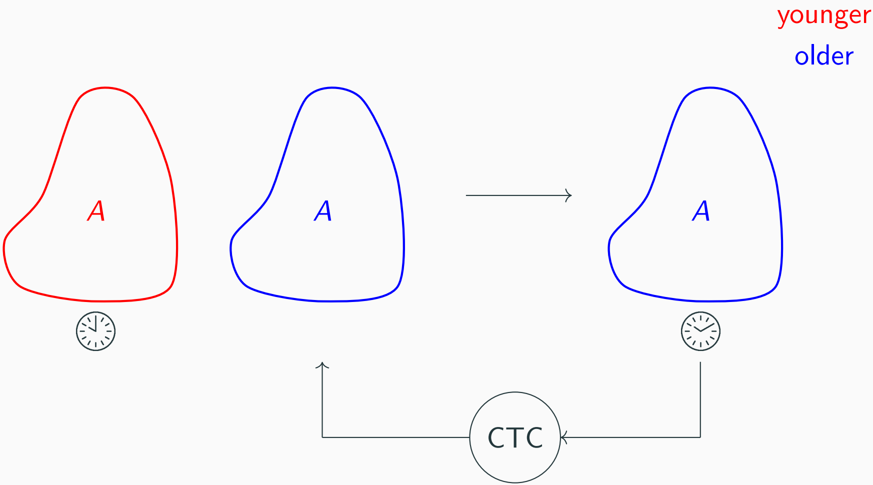

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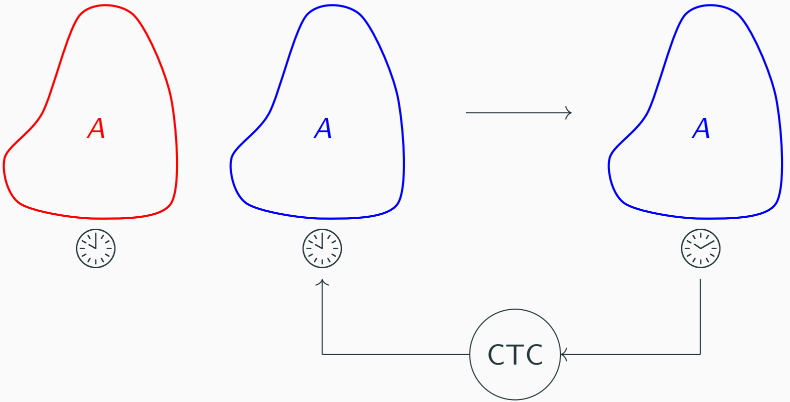

General CTC Mechanics

younger

older

much older

Now under normal time evolution (no interactions)

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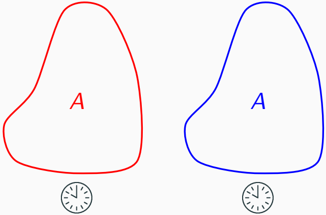

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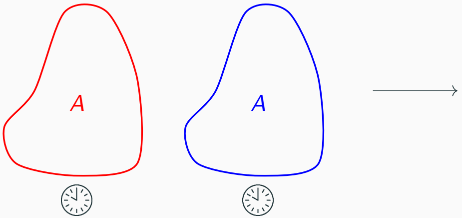

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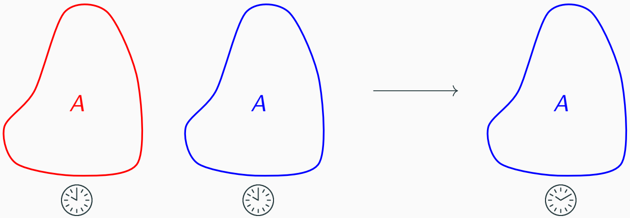

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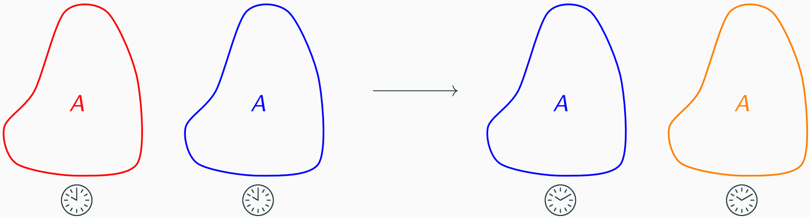

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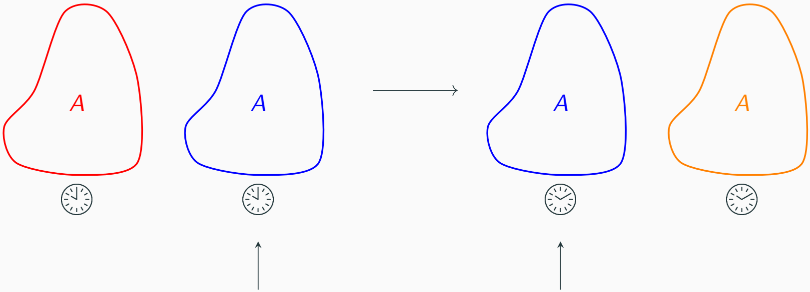

the older states are the same!

General CTC Mechanics

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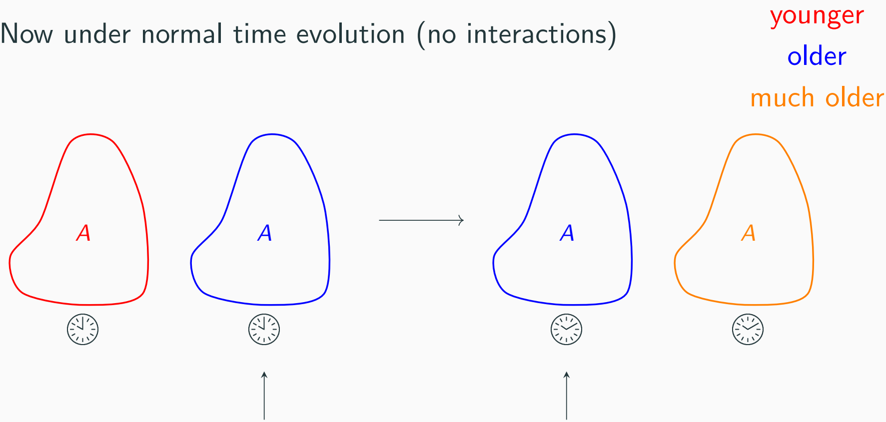

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Is this true if there are interactions?

General CTC Mechanics

Now under normal time evolution (no interactions)

younger
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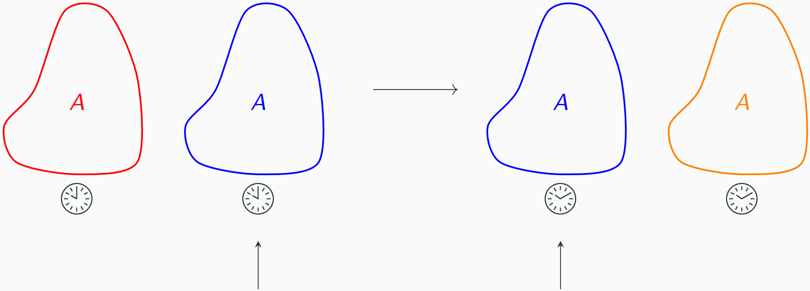

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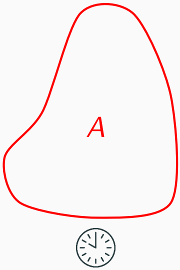
Is this true if there are interactions? (Yes!)

General CTC Mechanics

younger

older

Let U be an interaction that acts as follows



(Blob shape = state of system)

General CTC Mechanics

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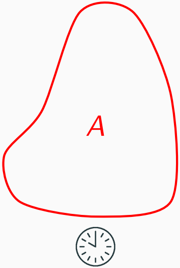

$$\xrightarrow{U}$$

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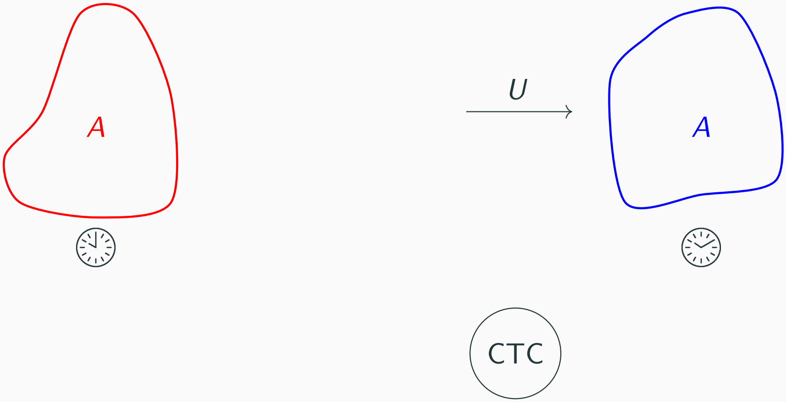

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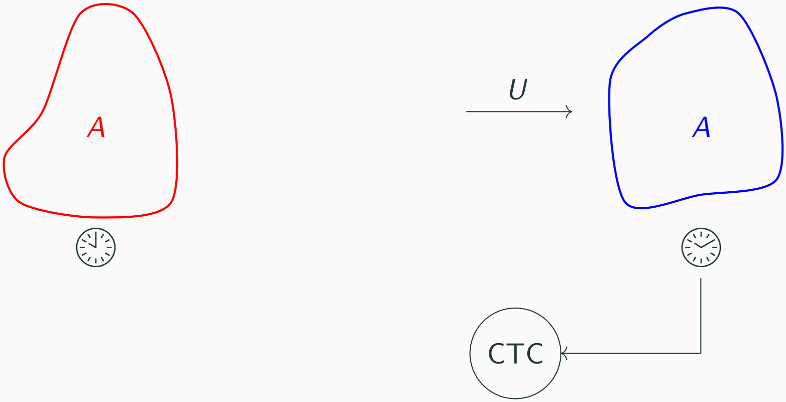

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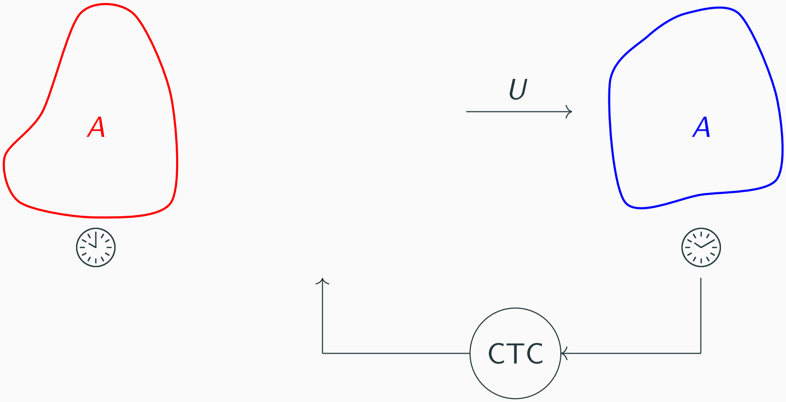

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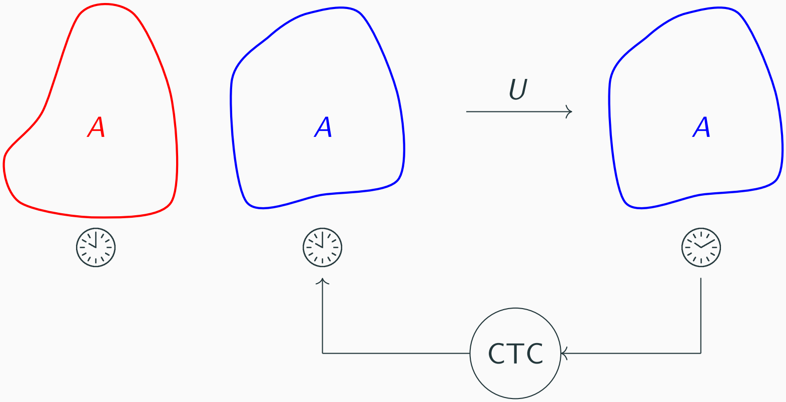

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General CTC Mechanics

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Now under normal time evolution (with interaction U)

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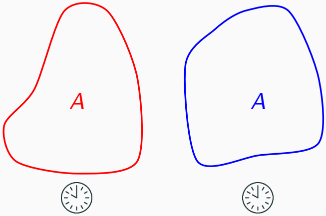

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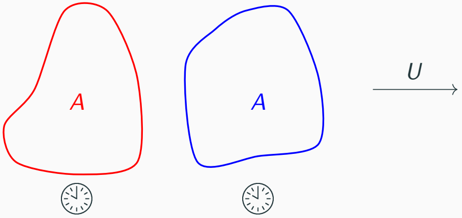

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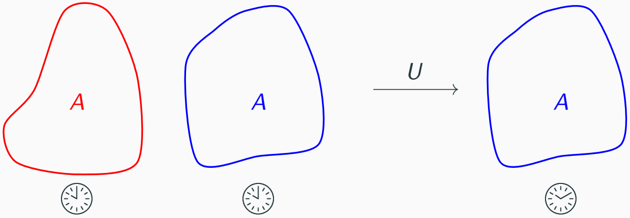

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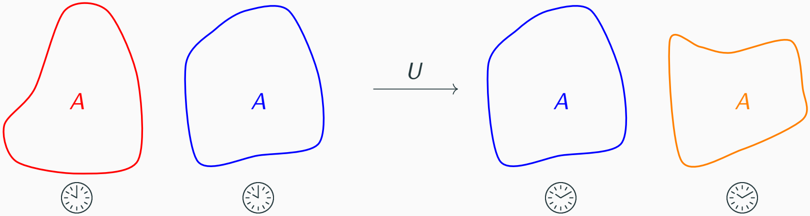

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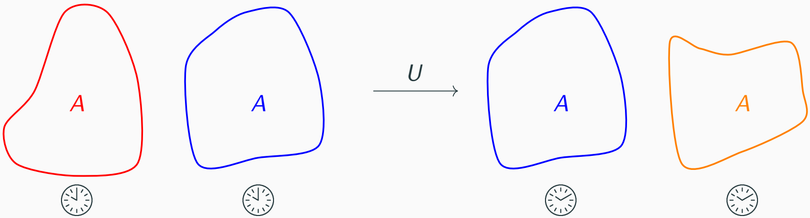

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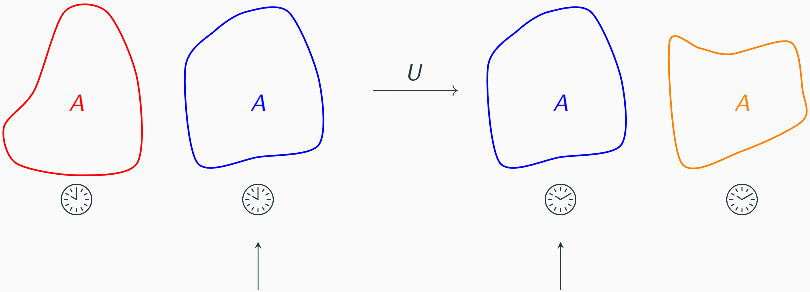
Thus, even with an interaction U,

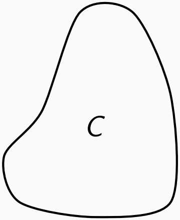


older states are *always* the same!

Classical Computation with CTCs (Paradoxes!)

Classical Computation with CTCs

Let C be a **classical** system



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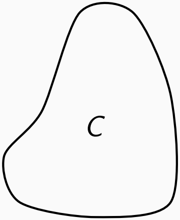

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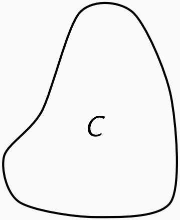

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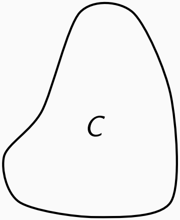

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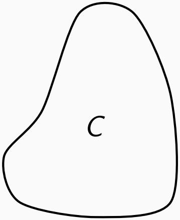

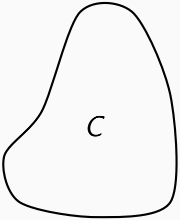

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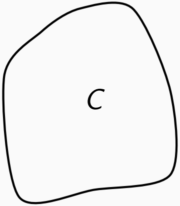

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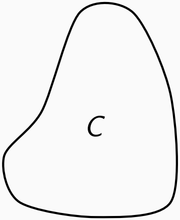

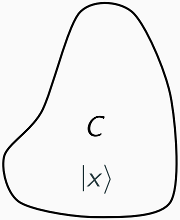

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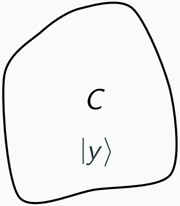

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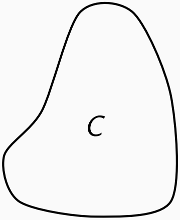

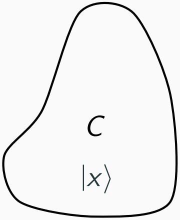

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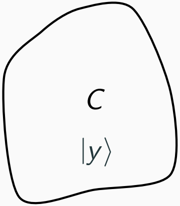
$x, y \in \{0, 1\}$



Paradox 1: Constraint on Initial Data

Consider the interaction $U : |x\rangle|y\rangle \longmapsto |x \oplus y\rangle\, |y\rangle$

younger

older

much older

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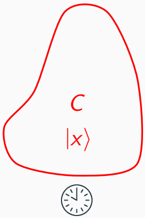

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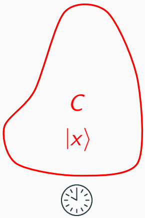

$\xrightarrow{U}$

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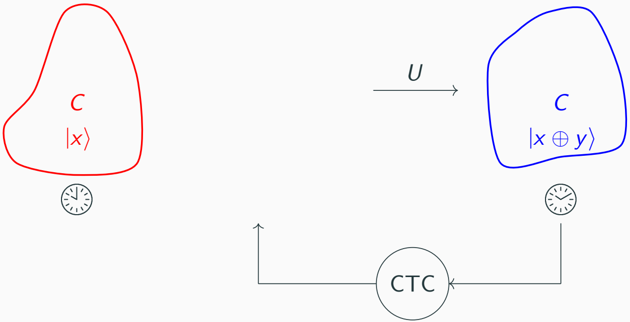

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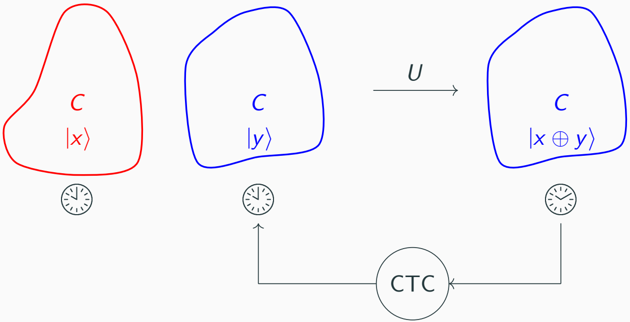

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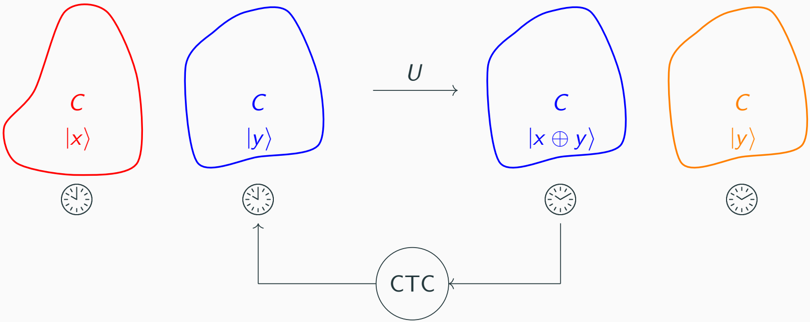

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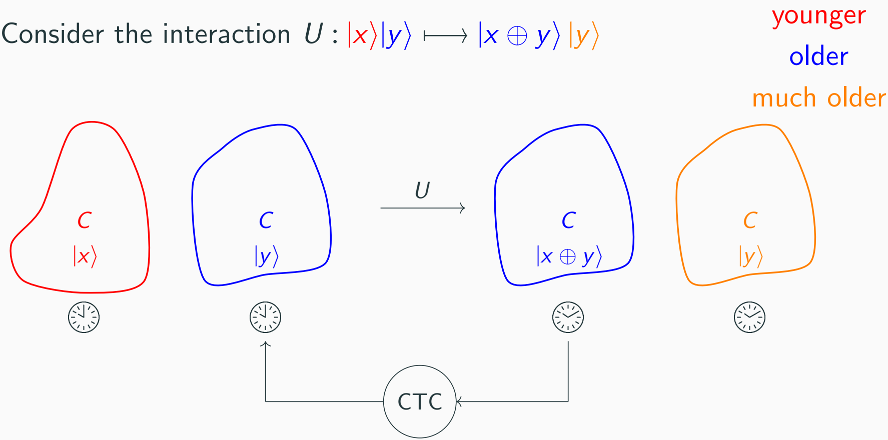

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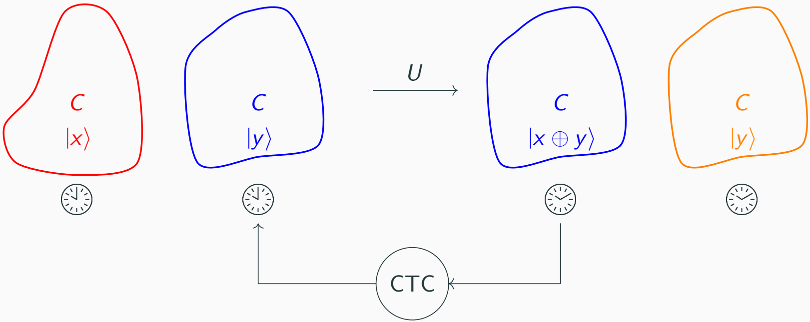

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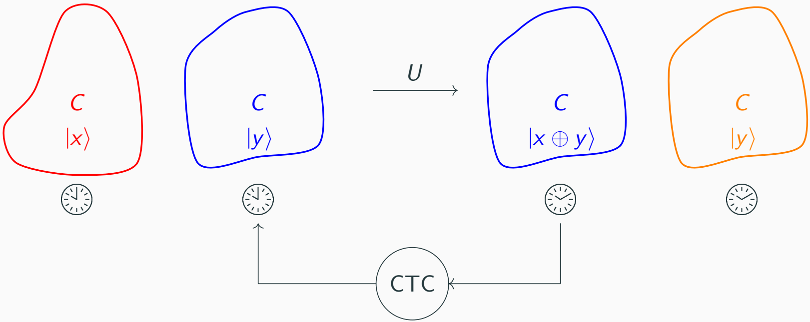

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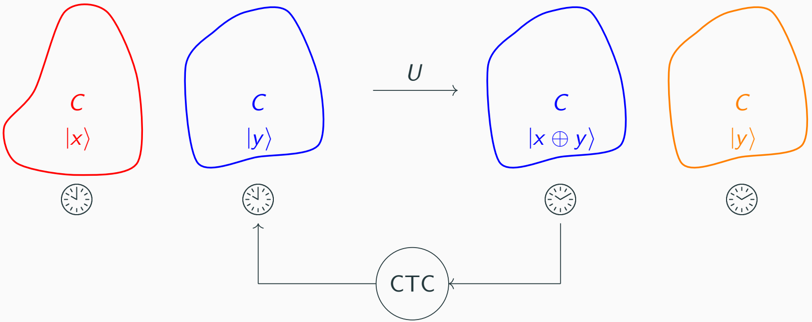

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This interaction allowed if and only if initial state of C is $|0\rangle$

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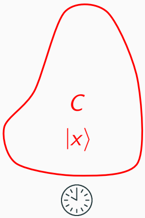

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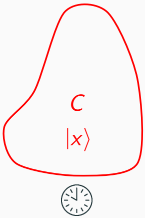

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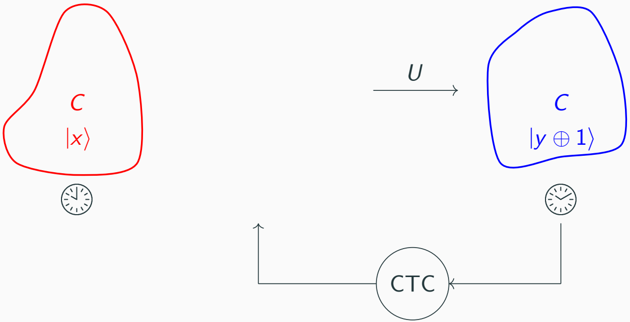

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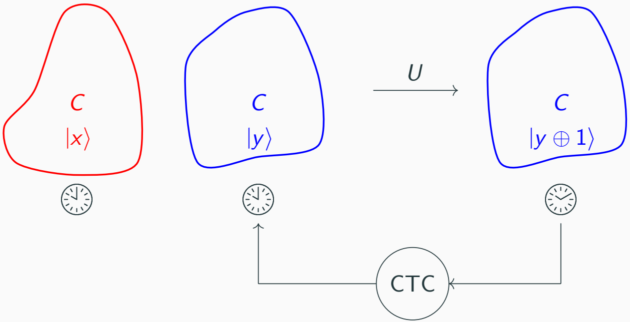

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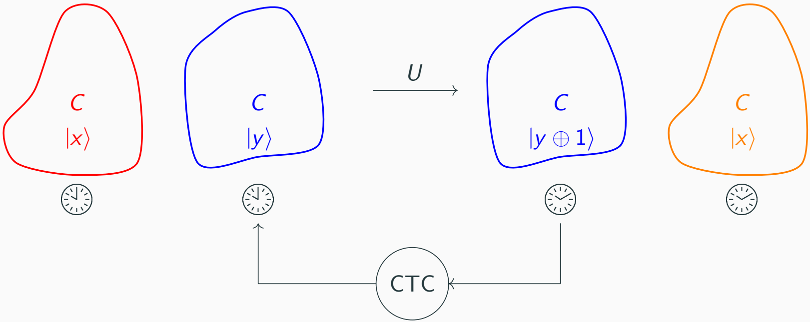

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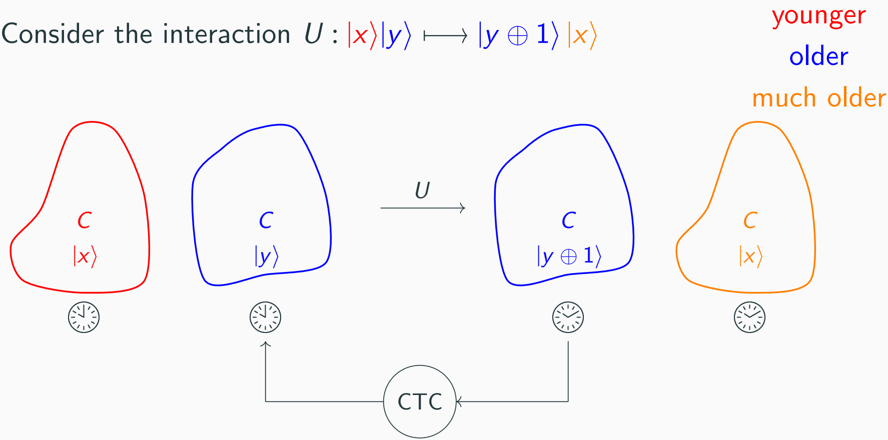

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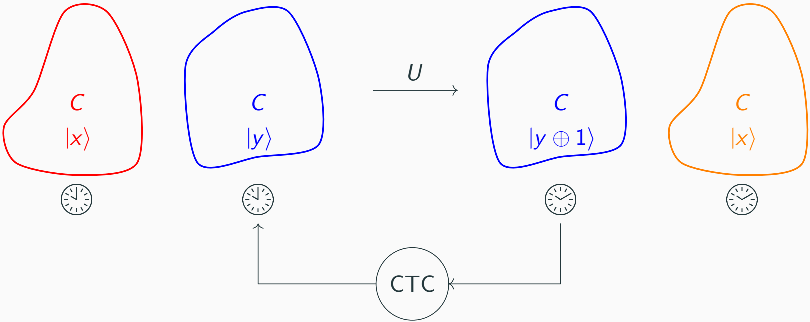

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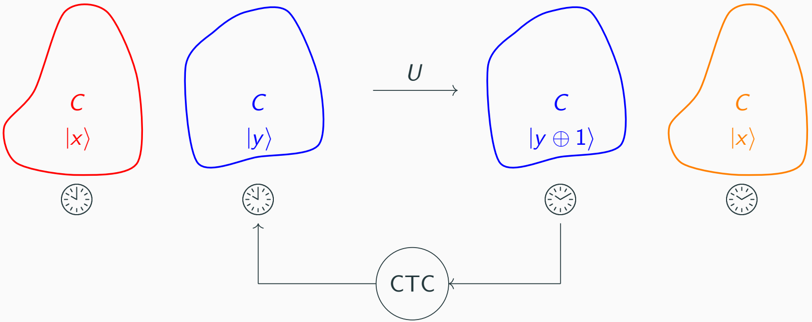

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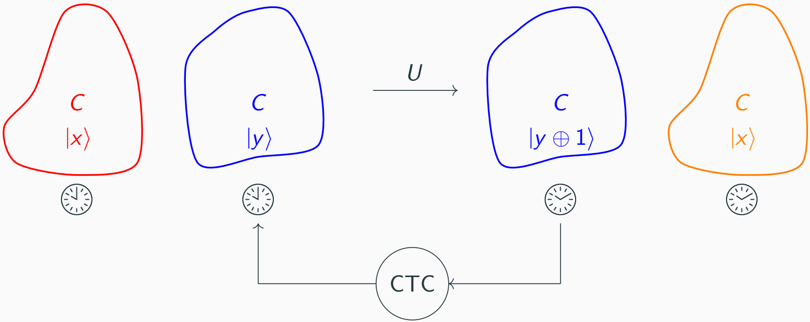

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States subject to U when traveling in time do not travel in time

Change of Representation: Fock Basis

Define state of $C = |\text{particle } \#\rangle$

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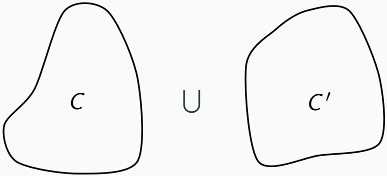
Let $\bar{C}$ be the composite **classical** system

$\bar{C} =$



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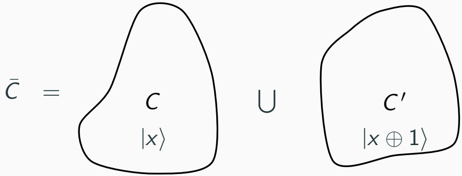

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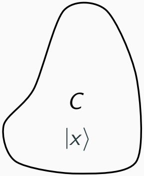
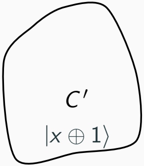
$\bar{C} \;=$  $\bigcup$ 

C and C' are classical systems containing either 0 or 1 particle(s)

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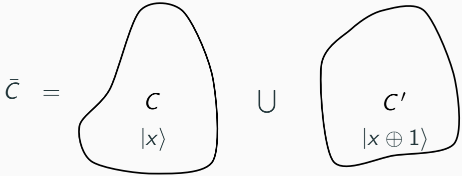

C and C' are classical systems containing either 0 or 1 particle(s)

State of $\bar{C}$ is $|x\rangle|x \oplus 1\rangle \in \{|0\rangle|1\rangle, |1\rangle|0\rangle\}$, so $\bar{C}$ contains 1 particle

Paradox 3: The Grandfather Paradox

Define C and C' as follows

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$$C = \text{system will } \textbf{not} \text{ traverse CTC in the future}$$
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Paradox 3: The Grandfather Paradox

Define C and C' as follows

$$\begin{aligned} C &= \text{system will } \textbf{not} \text{ traverse CTC in the future} \\ C' &= \text{system will traverse CTC in the future} \end{aligned}$$

Question: Does a particle ever traverse the CTC?

Paradox 3: The Grandfather Paradox

Run $\bar{C}$ through the CTC with interaction

Paradox 3: The Grandfather Paradox

younger
older
much older

Run $\bar{C}$ through the CTC with interaction

$$U : |x\rangle|x \oplus 1\rangle|y\rangle \longmapsto |x \oplus y\rangle|x \oplus y \oplus 1\rangle|y\rangle$$

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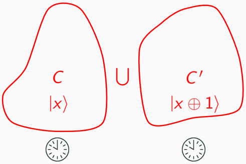

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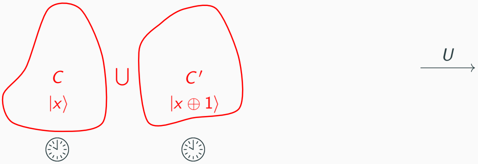

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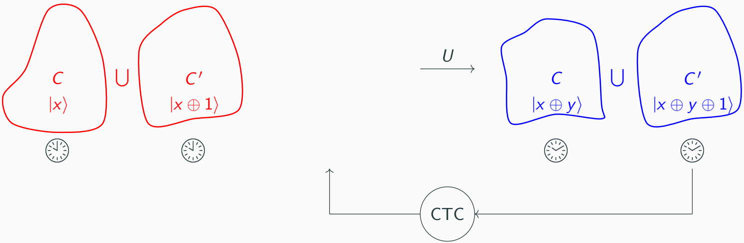

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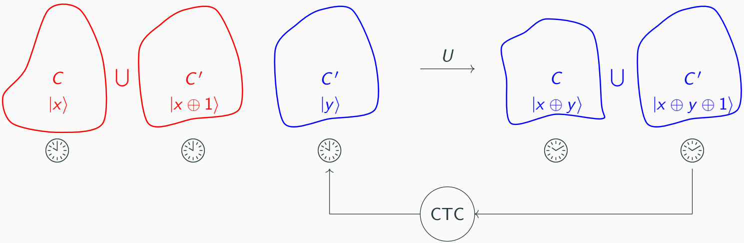

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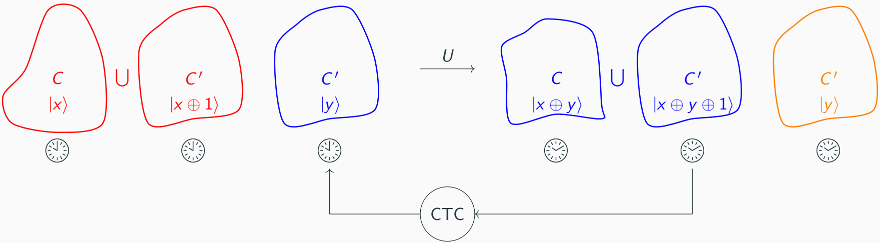

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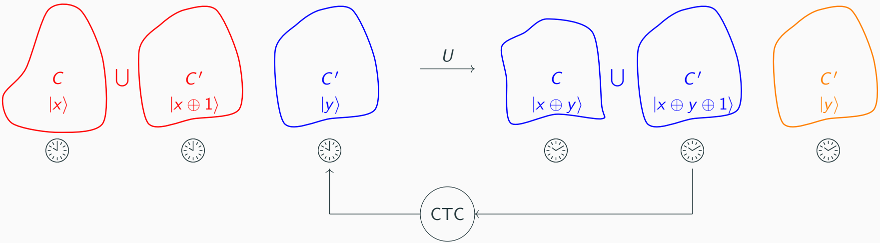

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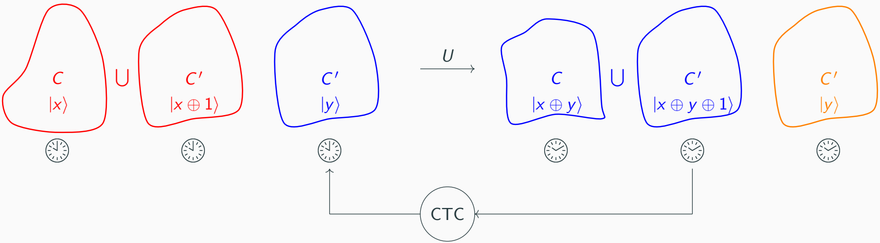

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$$y = x \oplus y \oplus 1$$

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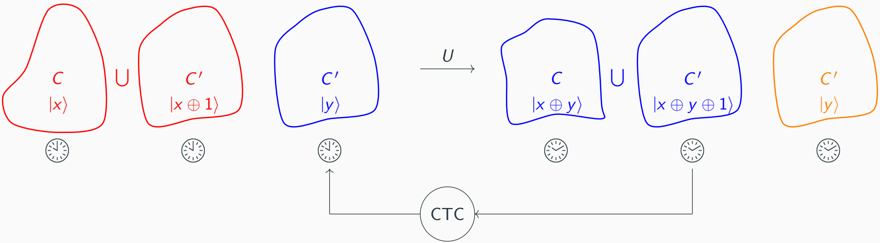

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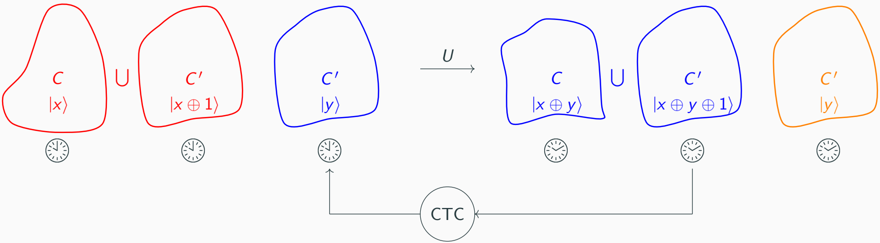
Run $\bar{C}$ through the CTC with interaction

$$U : |x\rangle|x \oplus 1\rangle|y\rangle \longmapsto |x \oplus y\rangle|x \oplus y \oplus 1\rangle|y\rangle$$



Older states must be the same

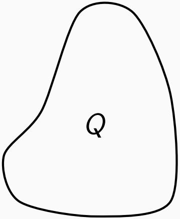
$$y = x \oplus y \oplus 1 \implies x = 1$$

Hence state of $\bar{C}$ is $|1\rangle|0\rangle$, so no particle traverses the CTC

Deutschian CTCs (D-CTCs)

Quantum Computation with CTCs

Let Q be a **quantum** system



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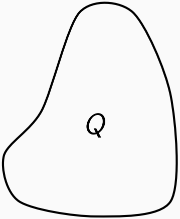

$\mathcal{H}$ = possible states of Q

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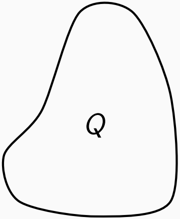

$$\mathcal{H} = \text{possible states of } Q$$

quantum $\implies$ $\mathcal{H} = \{\alpha\,|0\rangle + \beta\,|1\rangle \;:\; |\alpha|^2 + |\beta|^2 = 1\}$

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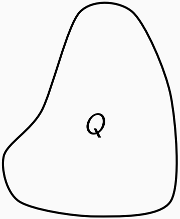

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$$\textbf{quantum} \quad \Longrightarrow \quad \mathcal{H} = \{\alpha\,|0\rangle + \beta\,|1\rangle \;:\; |\alpha|^2 + |\beta|^2 = 1\}$$

(superpositions!)

Deutschian CTCs (D-CTCs)

From before, even with a quantum system Q,

Deutschian CTCs (D-CTCs)

younger

older

much older

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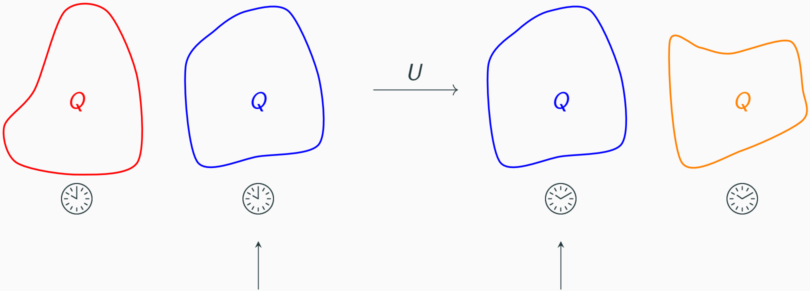

older states are *always* the same!

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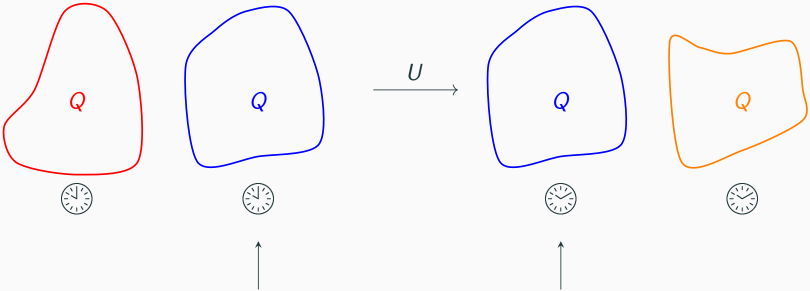

older states are *always* the same!

Question: How to frame this condition quantum mechanically?

Deutschian CTCs (D-CTCs)

Useful tool: density matrices!

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$$\rho = \text{state of system } Q$$

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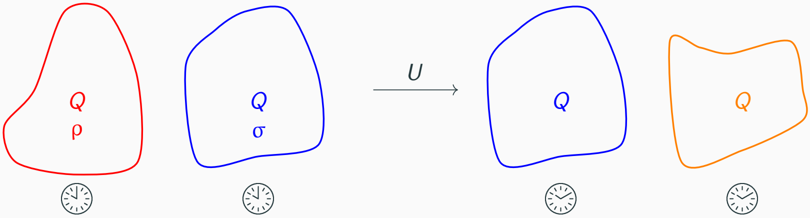
Now under a (unitary) interaction U and CTC traversal



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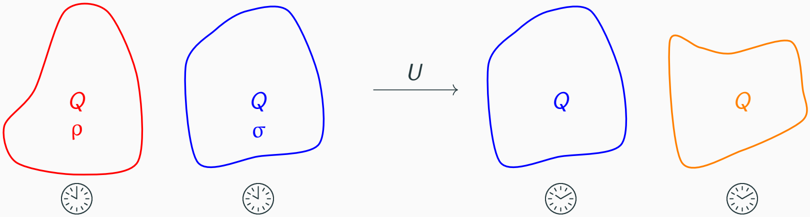
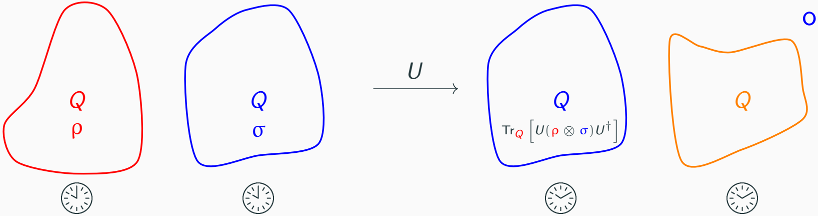


$$U : \rho \otimes \sigma \longmapsto U(\rho \otimes \sigma)U^\dagger$$

Deutsch's Consistency Condition (DCC)

younger

older



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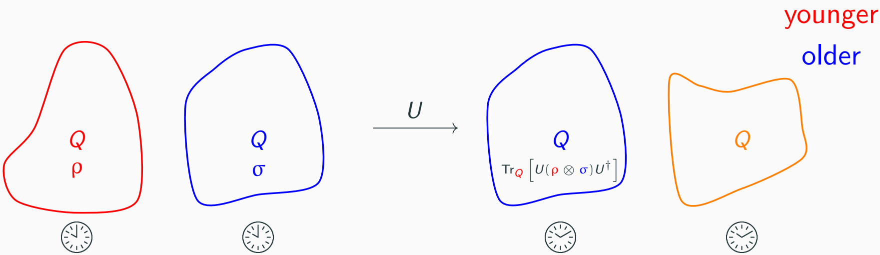

$$U : \rho \otimes \sigma \longmapsto U(\rho \otimes \sigma)U^\dagger$$

The state of system Q after U and CTC is then

$$\mathrm{Tr}_Q\left[U(\rho \otimes \sigma)U^\dagger\right]$$

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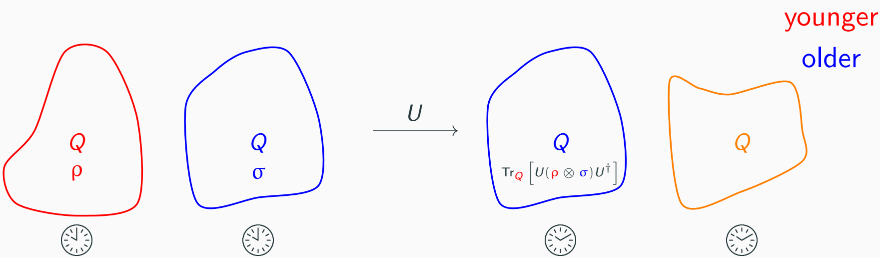

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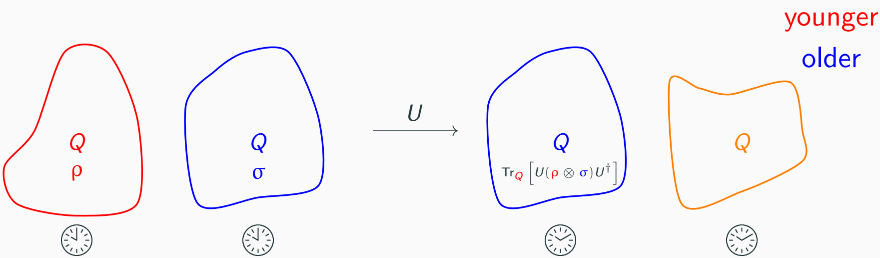

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$$\sigma = \mathrm{Tr}_Q\left[U(\rho \otimes \sigma)U^\dagger\right] \qquad \text{(for all } U\text{)}$$

Deutsch's Consistency Condition (DCC)

younger

older

Deutsch's consistency condition (DCC)

$$\sigma = \mathrm{Tr}_Q \left[U(\rho \otimes \sigma) U^\dagger \right] \tag{1}$$

Deutsch's Consistency Condition (DCC)

younger
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Deutsch's Consistency Condition (DCC)

younger

older

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Important Question: Does (1) constrain ρ?

Deutsch's Consistency Condition (DCC)

younger

older

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Deutsch's Answer:

Deutsch's Consistency Condition (DCC)

younger

older

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Important Question: Does (1) constrain ρ?

Deutsch's Answer: No!

Deutsch's Consistency Condition (DCC)

younger
older

Proof Idea

D. Deutsch, *Phys. Rev. D* **44**, 10, 3197 (1991)

younger
older

Proof Idea

Define an operator S by

$$S(\star) = \mathrm{Tr}_{Q}\left[U(\rho \otimes \star)U^{\dagger}\right]$$

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Question: Does D-CTC model resolve classical paradoxes?

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Resolution to Paradox 1

$$U : |x\rangle|y\rangle \longmapsto |x \oplus y\rangle\, |y\rangle$$

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Resolution to Paradox 1

younger

older

much older

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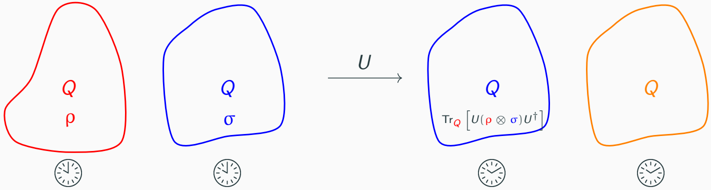

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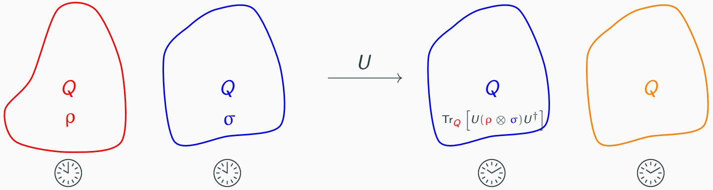

DCC ultimately implies

$$\rho = \frac{1}{2}\mathbf{I} + \left(\gamma\,|0\rangle\,\langle 1| + \gamma^*\,|1\rangle\,\langle 0|\right) \quad \left(0 \le |\gamma|^2 \le 1\right)$$

Resolution to Paradox 2

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Resolution to Paradox 2

younger

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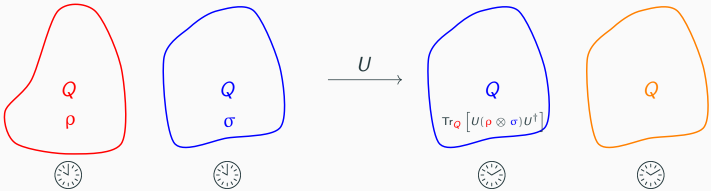

Resolution to Paradox 2

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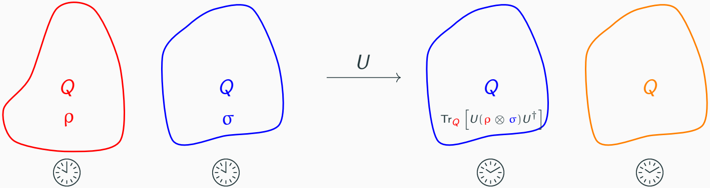

DCC ultimately implies

$$\rho = \frac{1}{2}\mathbf{I} + \frac{\lambda}{2}\left(|0\rangle\langle 1| + |1\rangle\langle 0|\right) \qquad (0 \le \lambda \le 1)$$

Resolution to Paradox 3

$$U : |x\rangle|x \oplus 1\rangle|y\rangle \longmapsto |x \oplus y\rangle|x \oplus y \oplus 1\rangle|y\rangle$$

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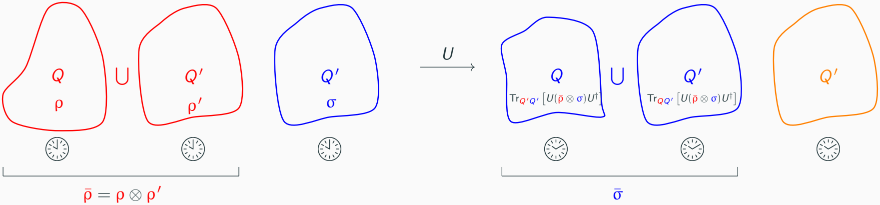

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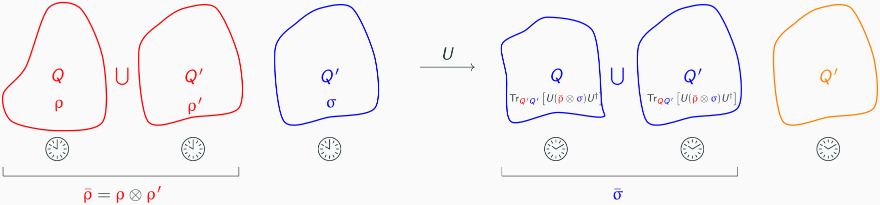

If initial state of $\bar{Q}$ is the classically forbidden $|0\rangle\,|1\rangle$, DCC implies

$$\rho = \frac{1}{2}\mathbf{I}$$

Resolution to Paradox 3

For initial state $|0\rangle|1\rangle = |01\rangle$, DCC also implies older state of $\bar{Q}$ is

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For initial state $|0\rangle|1\rangle = |01\rangle$, DCC also implies older state of $\bar{Q}$ is

$$\bar{\sigma} = \frac{1}{2}\left(|00\rangle\langle 00| + |11\rangle\langle 11|\right)$$

Quantum Computation with D-CTCs

Quantum State Cloning

D. Ahn, T. C. Ralph, and R. B. Mann, arXiv:1008.0221 [Gr-Qc, Physics:Hep-Th, Physics:Quant-Ph] (2010)

D. Bacon, *Phys Rev. A* **70**, 032309 (2004)

Quantum Implications of D-CTCs

Quantum State Cloning

Given quantum states $|\psi\rangle, |\phi\rangle$, there exists procedure U such that

$$U : |\psi\rangle |\phi\rangle \longrightarrow |\psi\rangle |\psi\rangle$$

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Insecure Quantum Cryptography

BB84 and B92 insecure. Entanglement-based procedures still safe.

D. Ahn, T. C. Ralph, and R. B. Mann, arXiv:1008.0221 [Gr-Qc, Physics:Hep-Th, Physics:Quant-Ph] (2010)

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Implications of D-CTCs to Quantum Computation

Recall,

S. Aaronson and J. Watrous, *Proc. R. Soc. A* **465**, 631 (2008)

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Recall,

$$\mathbf{P} = \text{decision problems solvable in polynomial time}$$

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Aaronson and Watrous proved the astounding result

$$\mathbf{P}_{\mathsf{CTC}} \supseteq \mathbf{NP}$$

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PSPACE = decision problems solvable in polynomial space (unbounded time)

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Recall,

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S. Aaronson and J. Watrous, *Proc. R. Soc. A* **465**, 631 (2008)

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Recall,

BQP = decision problems solvable with high probability in polynomial time

S. Aaronson and J. Watrous, *Proc. R. Soc. A* **465**, 631 (2008)

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Main Punchline

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Corollary

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Corollary

Computational advantages of quantum computers over classical computers are a function of space-time (since CTCs manifest from space-time curvature)

References

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