Quantum-Classical Gaps
&
Quantum Shallow Circuits

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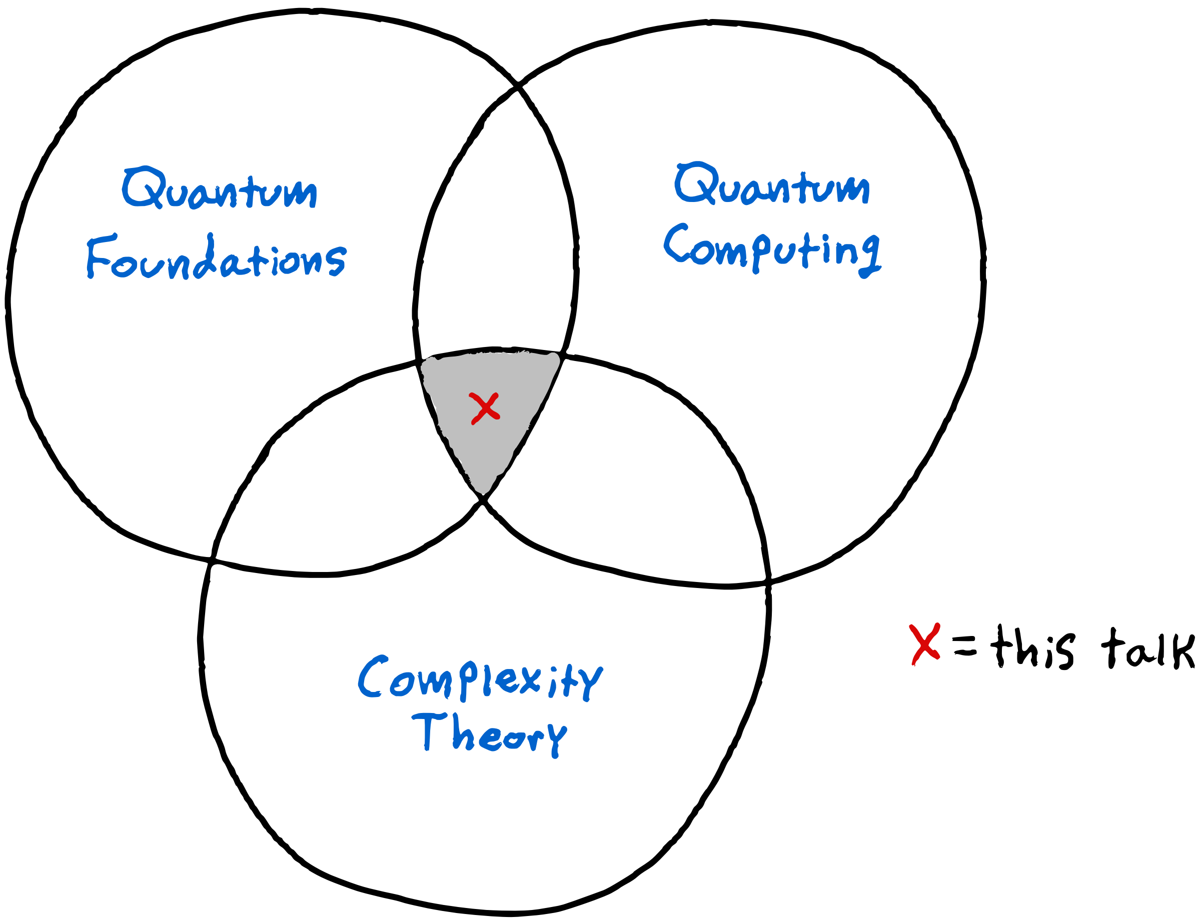

Quantum Foundations
Quantum Computing
Complexity Theory
x
x = this talk

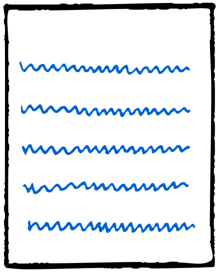
What Makes a "Quantum Theory" Quantum?

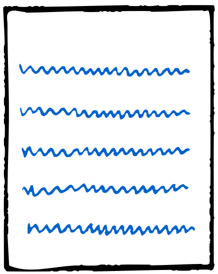
Canonically Quantum Phenomena = { interference, superposition, entanglement, no-cloning, teleportation, measurement, ...

$$\boxed{\text{Canonically Quantum} \neq \text{Quantum}}$$

arxiv:quant-ph/0401052

Operational Quantum Mechanics

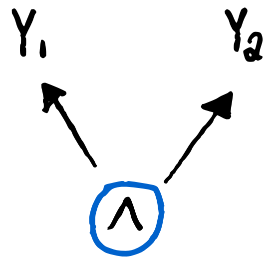
P =  $\longleftrightarrow$ ρ_P

M =  $\longleftrightarrow$ $\{E_k^M\}$

$$\boxed{\Pr(k \mid P, M) = \mathrm{Tr}(\rho_P E_k^M)}$$

The EPR Scenario

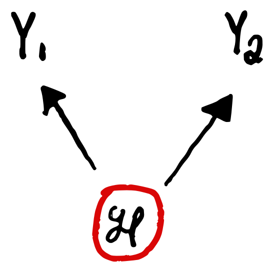
EPR



$\underline{Pr(Y_1, Y_2):}$

$$\sum_{\Lambda=\lambda} Pr(Y_1, Y_2 \mid \lambda)\, Pr(\lambda)$$

QEPR



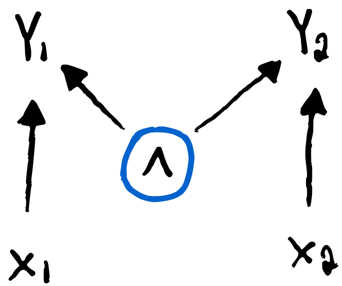
$$Tr\left(M_{Y_1} \otimes M_{Y_2}\, \rho_{12}\right)$$

Not quintessentially quantum!

$$\boxed{\mathcal{P}(\text{EPR}) = \mathcal{P}(\text{QEPR})}$$

The Bell Scenario

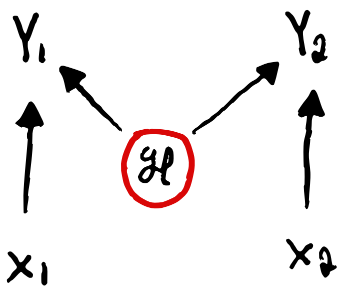
Bell



$Pr(Y_1, Y_2 \mid X_1, X_2)$:

$$\sum_{\Lambda=\lambda} Pr(Y_1 \mid X_1, \lambda) Pr(Y_2 \mid X_2, \lambda) \times Pr(X_1) Pr(X_2) Pr(\lambda)$$

QBell



$$Tr(M_{Y_1|X_1} \otimes M_{Y_2|X_2} \rho_{12})$$

Quintessentially Quantum!

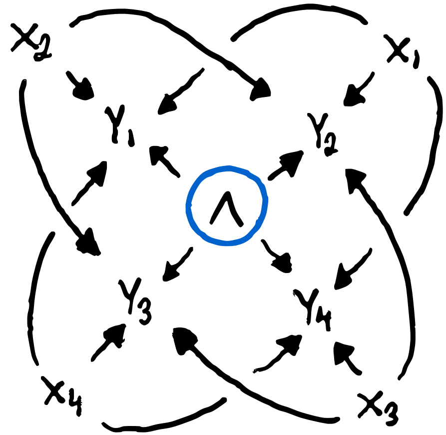
$$\boxed{\mathcal{P}(\text{Bell}) \subsetneq \mathcal{P}(\text{QBell})}$$ (Bell's theorem)

The Generalized GHZ Scenario

$GHZ_{n,m}$ = "n Parties + m nearest-neighbor Communication"

For example:

$GHZ_{4,2}$ =



Quintessentially Quantum!

$$\boxed{\mathcal{P}(GHZ_{n,m}) \subsetneq \mathcal{P}(QGHZ_{n,m})} \qquad (m < n-1)$$

Quantum-Classical Gaps (QC-Gaps)

$G = (V \cup L_c, E)$

$QG = (V \cup \tilde{L}_c \cup L_Q, E)$

Directed Acyclic Graph (DAG)

Definition:

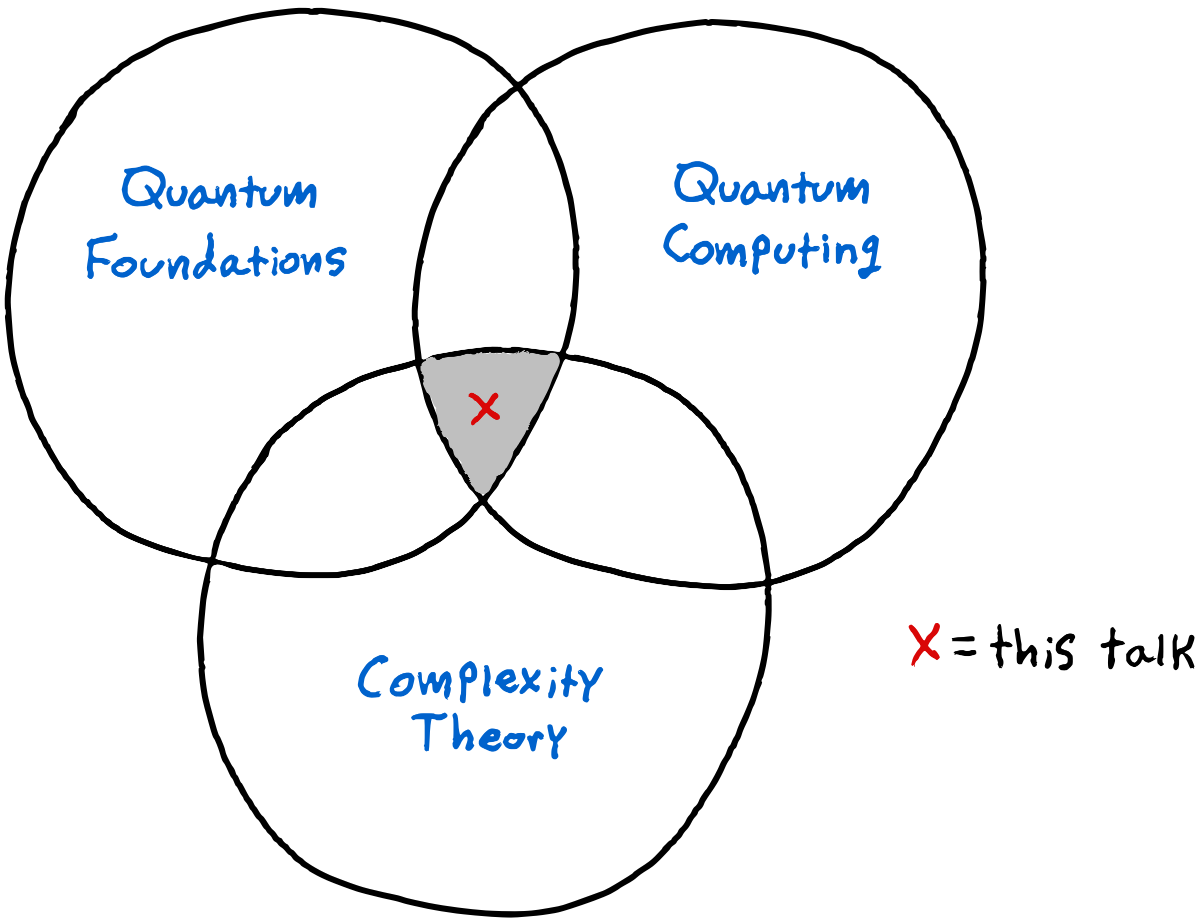
G admits QC-gap $\longleftrightarrow$ $\mathcal{P}(G) \subsetneq \mathcal{P}(QG)$

Theorem (Henson, Lal, Pusey; 2014):

G admits QC-gap $\longleftrightarrow$ $\exists$ Polynomial $\mathcal{B}$ such that:

(i) $\forall \mathcal{D} \in \mathcal{P}(G)$: "$\mathcal{B}(\mathcal{D}) \leq 0$"

(ii) $\exists \mathcal{D} \in \mathcal{P}(QG)$: "$\mathcal{B}(\mathcal{D}) > 0$"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Quantum Advantage & QC-Gaps

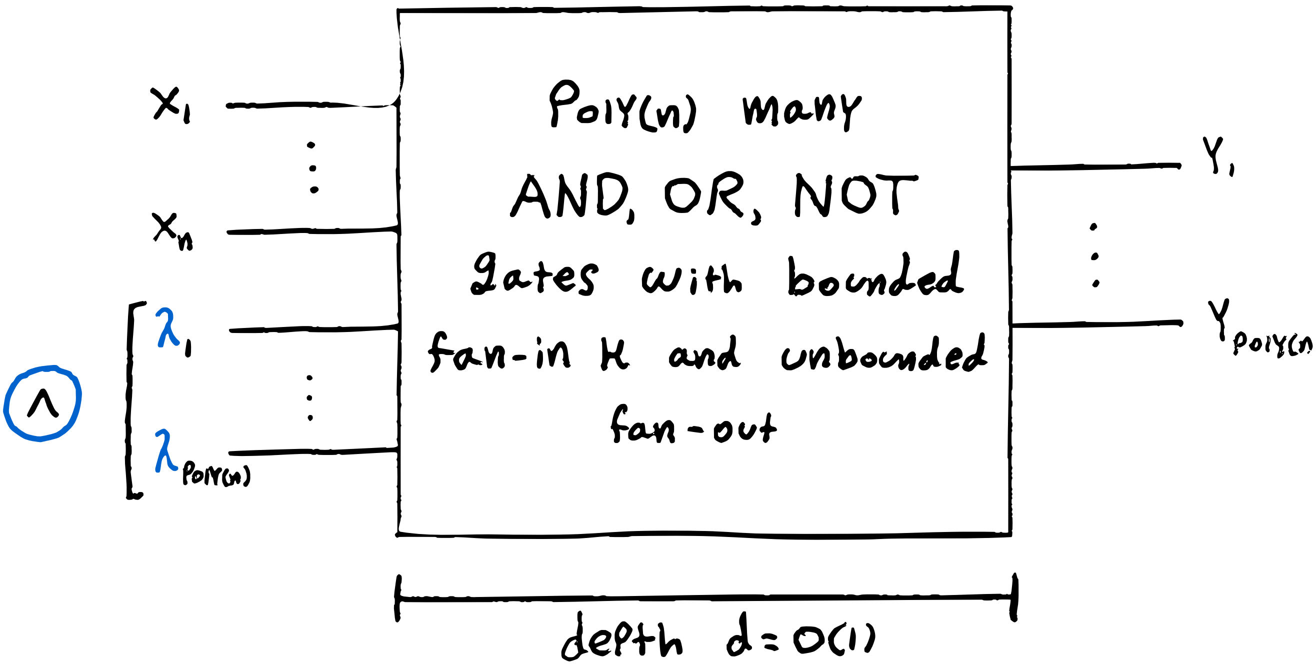
QC-Gap: $\mathcal{P}(G) \subsetneq \mathcal{P}(QG)$

Quantum advantage:

Classical Complexity Class $\subsetneq$ Quantum Complexity Class

Q: For a given computational model, can we prove or understand a quantum advantage using QC-gaps?

Classical Shallow Circuits



Observation:

$$|Pa(Y_i)| \leq k^d = O(1)$$

Complexity of Classical Shallow Circuits

Sampling Problem:

$$S = \bigcup_{x \in \{0,1\}^*} \mathcal{D}_x$$

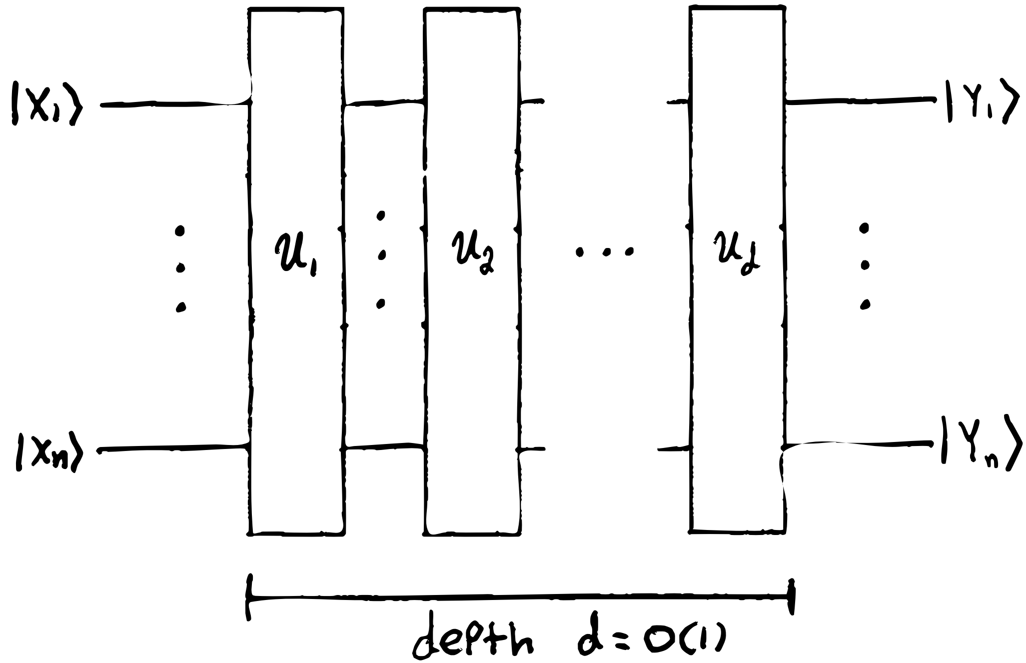
$$\text{SampNC}^0/\text{rpoly}_\varepsilon = \left\{ \begin{array}{l} \text{Sampling problems "Solvable" on} \\ \text{a Classical Shallow Circuit} \\ \text{within error } \varepsilon > 0 \end{array} \right\}$$

$$\text{SampNC}^0/\text{rpoly} = \bigcap_{\varepsilon > 0} \text{SampNC}^0/\text{rpoly}_\varepsilon$$

Theorem:

$$S \in \text{SampNC}^0/\text{rpoly} \longrightarrow \forall \mathcal{D}_x \in S : \mathcal{D}_x \in \mathcal{P}(\text{GHZ}_{\text{poly}(|x|),\, k^d})$$

Quantum Shallow Circuits



$$\mathcal{U}_i = \bigotimes_{j(i)} A_j, \qquad A_j \in \{H, S, T, CZ\}$$

Complexity of Quantum Shallow Circuits

$$\mathrm{SampQNC}^0 = \bigcap_{\varepsilon > 0} \mathrm{SampQNC}^0_\varepsilon$$

Theorem:

The distribution

Cluster State

$\{|0\rangle \pm e^{i\alpha}|1\rangle\}$ Projections

$$\Pr_{Y \sim \mathcal{D}}(Y) = \mathrm{Tr}\left(\rho_n \, M_1^{\alpha_1} \otimes \cdots \otimes M_n^{\alpha_n} \, |Y\rangle\langle Y|\right)$$

(i) is realizable on a quantum shallow circuit

(ii) Satisfies $\mathcal{D} \in \mathcal{P}(\mathrm{QGHZ}_{n,m})$

Quantum Advantage & QC-Gaps

Theorem (Bravyi, Gosset, König; 2018):

$$\mathsf{SampNC^0/rpoly} \subsetneq \mathsf{SampQNC^0}$$

Future Research:

Can we prove/understand this advantage with QC-gap:

$$\mathcal{P}(GHZ_{n,m}) \subsetneq \mathcal{P}(QGHZ_{n,m}) \qquad ?$$

Future Research

- Same question, but for search problems:

$$\mathsf{FNC^0/rpoly} \subsetneq \mathsf{FQNC^0} \quad ?$$

- "Search/sampling equivalence" on shallow circuits?
- Robust advantage and QC-gaps?
- Extend to more powerful computational models?

→ the holy grail:

$$\mathsf{SampBPP} \quad \text{vs.} \quad \mathsf{SampBQP} \quad ?$$

Thank you!

Credit: Javi

Tricks for Proving QC-Gaps

Definition:

A trick is a QC-gap non-increasing map:

$$(G, QG) \xrightarrow{*} (G^*, QG^*)$$

$$\rightarrow \mathcal{P}(QG) \setminus \mathcal{P}(G) \supseteq \mathcal{P}(QG^*) \setminus \mathcal{P}(G^*).$$

Theorem:

If G^* admits a QC-gap, then G admits a QC-gap.