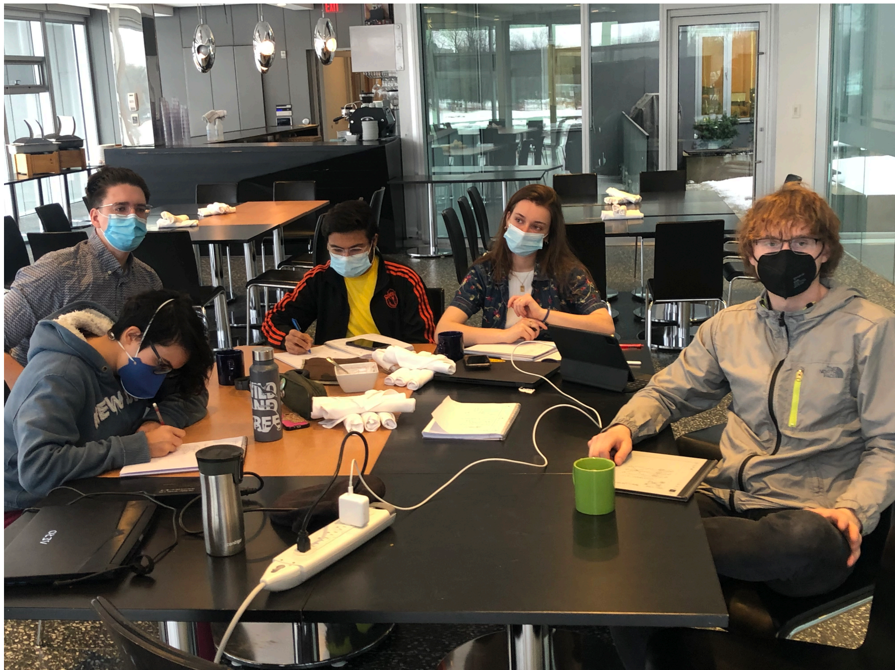
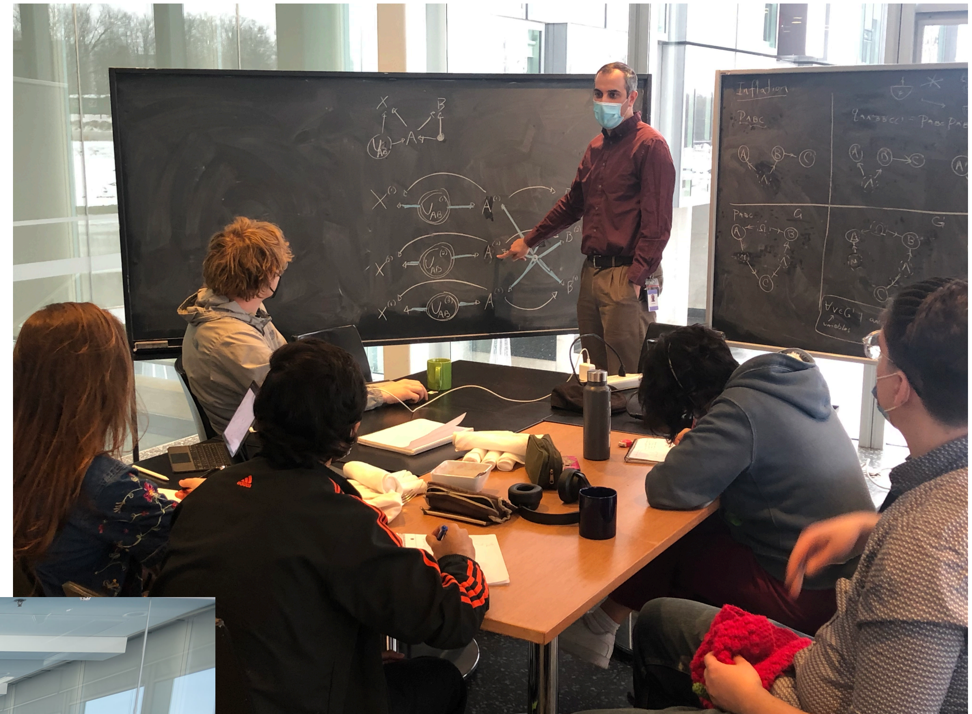
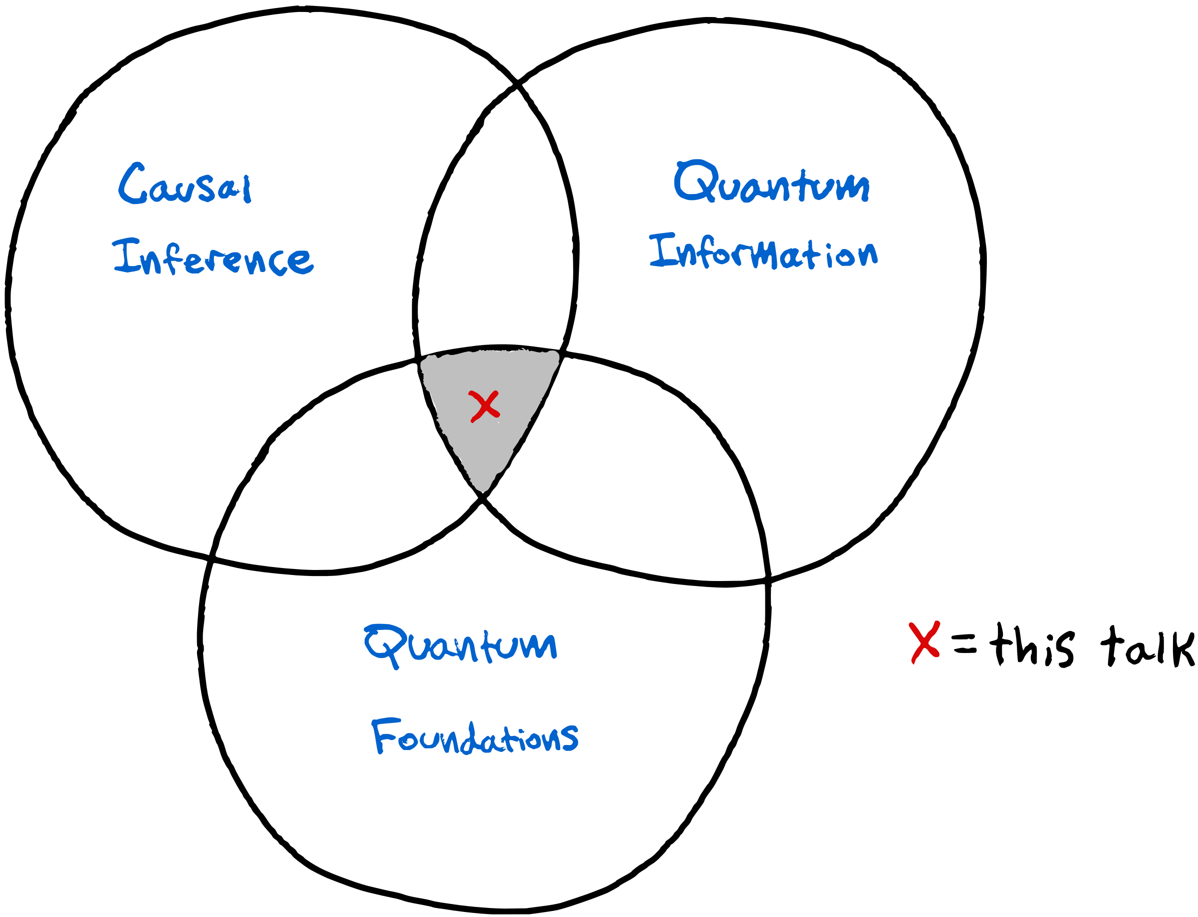
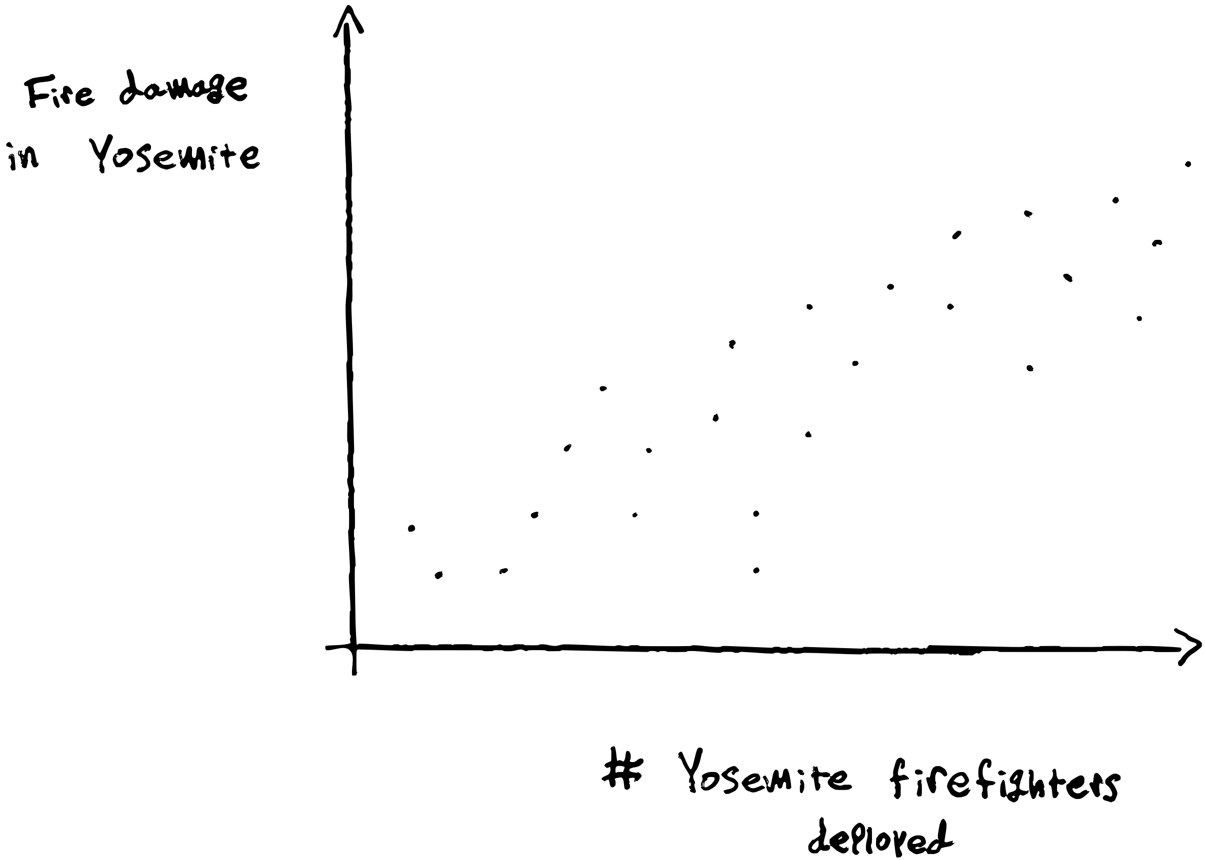
Quantum Causal Advantage is Contagious

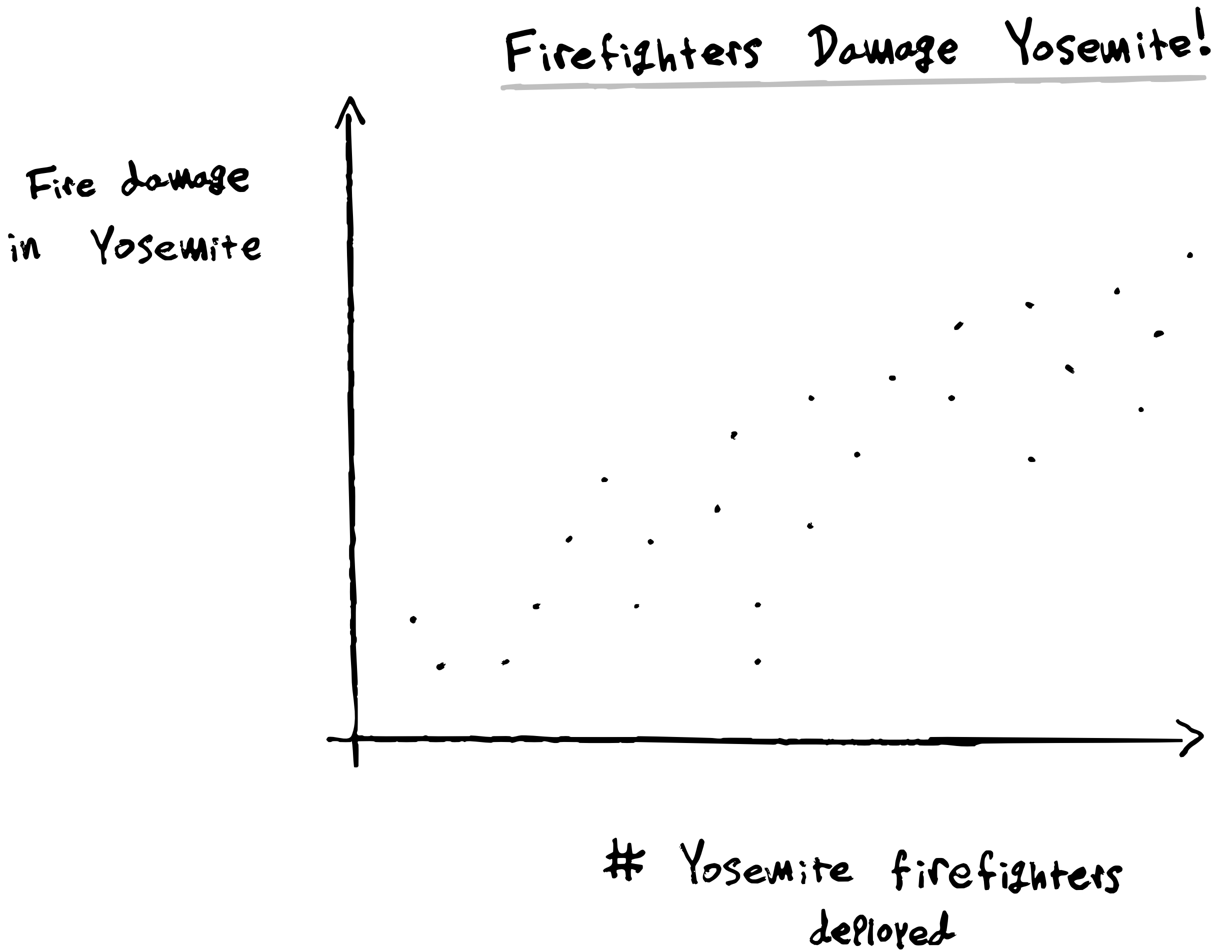
Matthew Fox



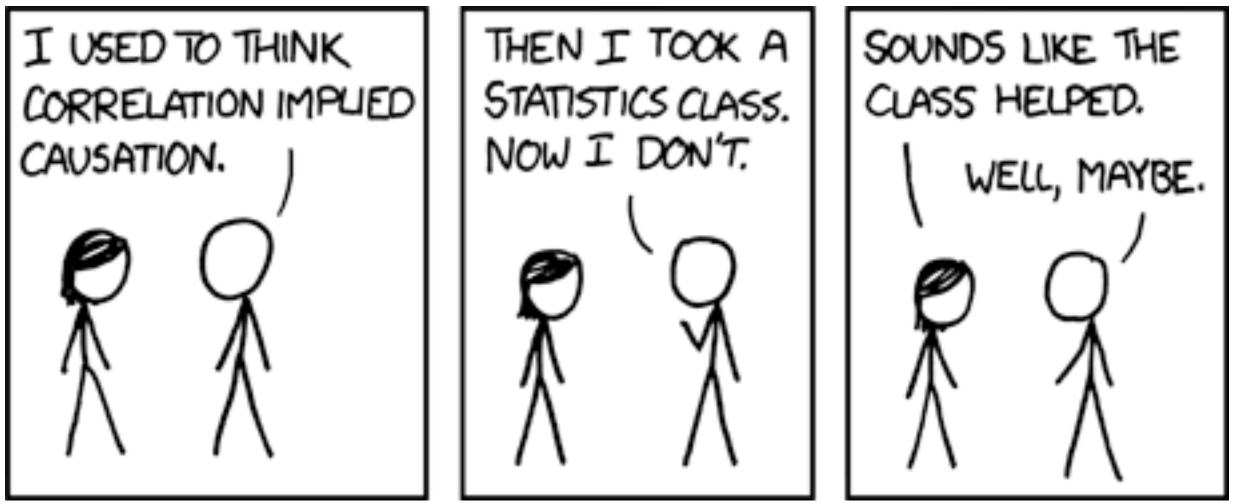



Causal Inference
Quantum Information
Quantum Foundations
X = this talk


Fire damage in Yosemite
Yosemite firefighters deployed

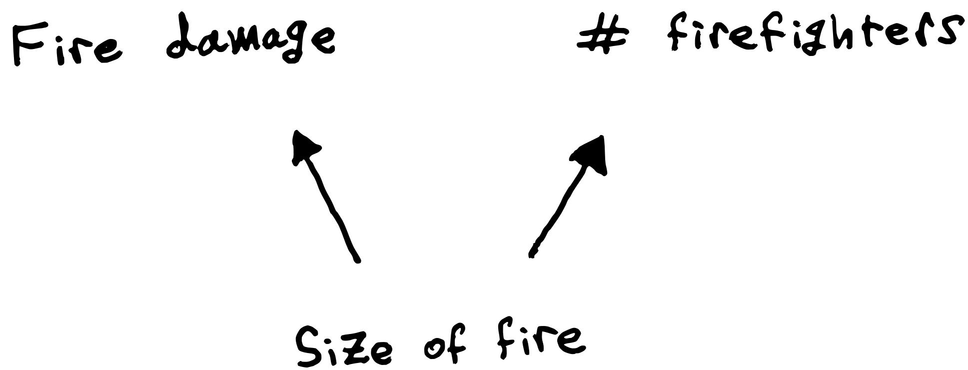

Firefighters Damage Yosemite!
Fire damage in Yosemite
Yosemite firefighters deployed

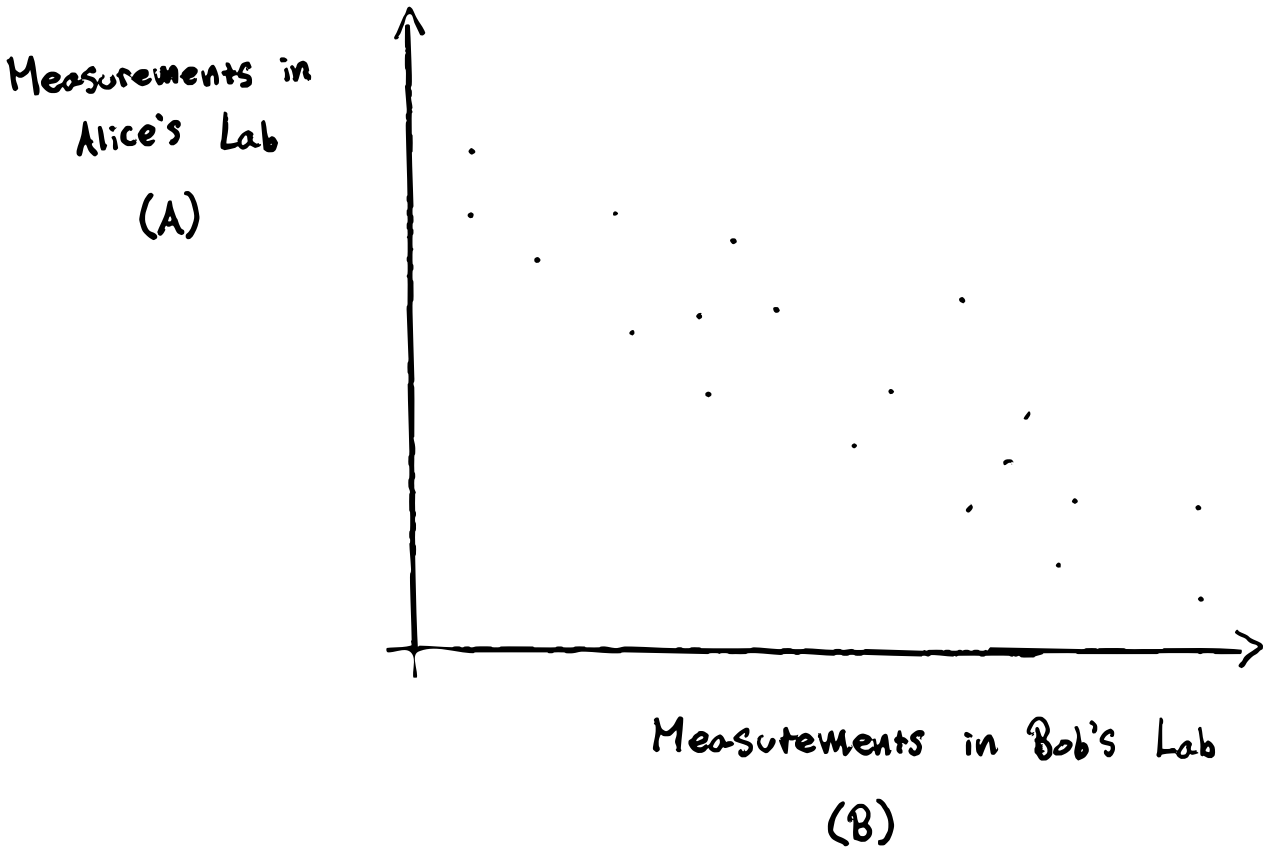
Correlation does not imply causation.



https://xkcd.com/552/

Reichenbach's Common Cause Principle




Measurements in Alice's Lab
(A)
Measurements in Bob's Lab
(B)

Reichenbach's Common Cause Principle

Q: If A, B relate causally, then how?

Possible Answers:

(i) $A \qquad B$

(ii) $A \longrightarrow B$

(iii) $A \longleftarrow B$

(iv)
$$\begin{tikzcd} A & & B \\ & \Lambda \arrow[ul] \arrow[ur] & \end{tikzcd}$$

$\vdots$

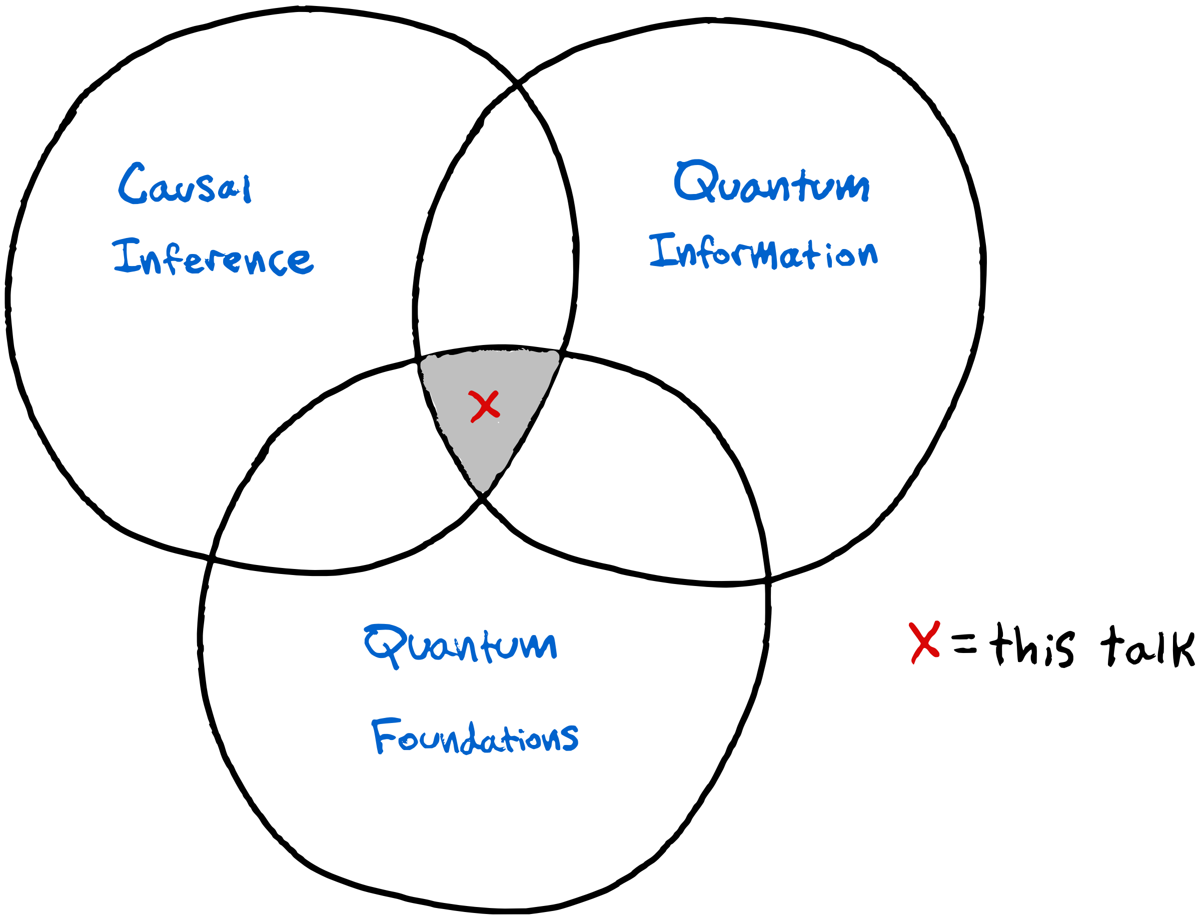
$P(A, B)$:

$$P(A)\,P(B)$$

$$P(B \mid A)\,P(A)$$

$$P(A \mid B)\,P(B)$$

$$\sum_{\Lambda} P(A \mid \Lambda)\,P(B \mid \Lambda)\,P(\Lambda)$$


Causal Inference
Quantum Information
Quantum Foundations
X
X = this talk

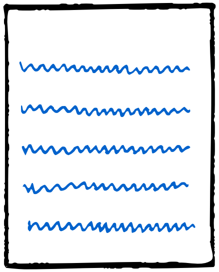
What Makes a "Quantum Theory" Quantum?

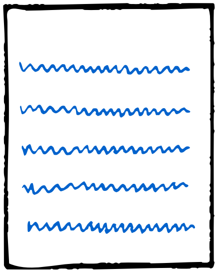
Canonically Quantum Phenomena $=$ {interference, superposition, entanglement, no-cloning, teleportation, measurement, ...}

$\boxed{\text{Canonically Quantum} \neq \text{Quantum}}$

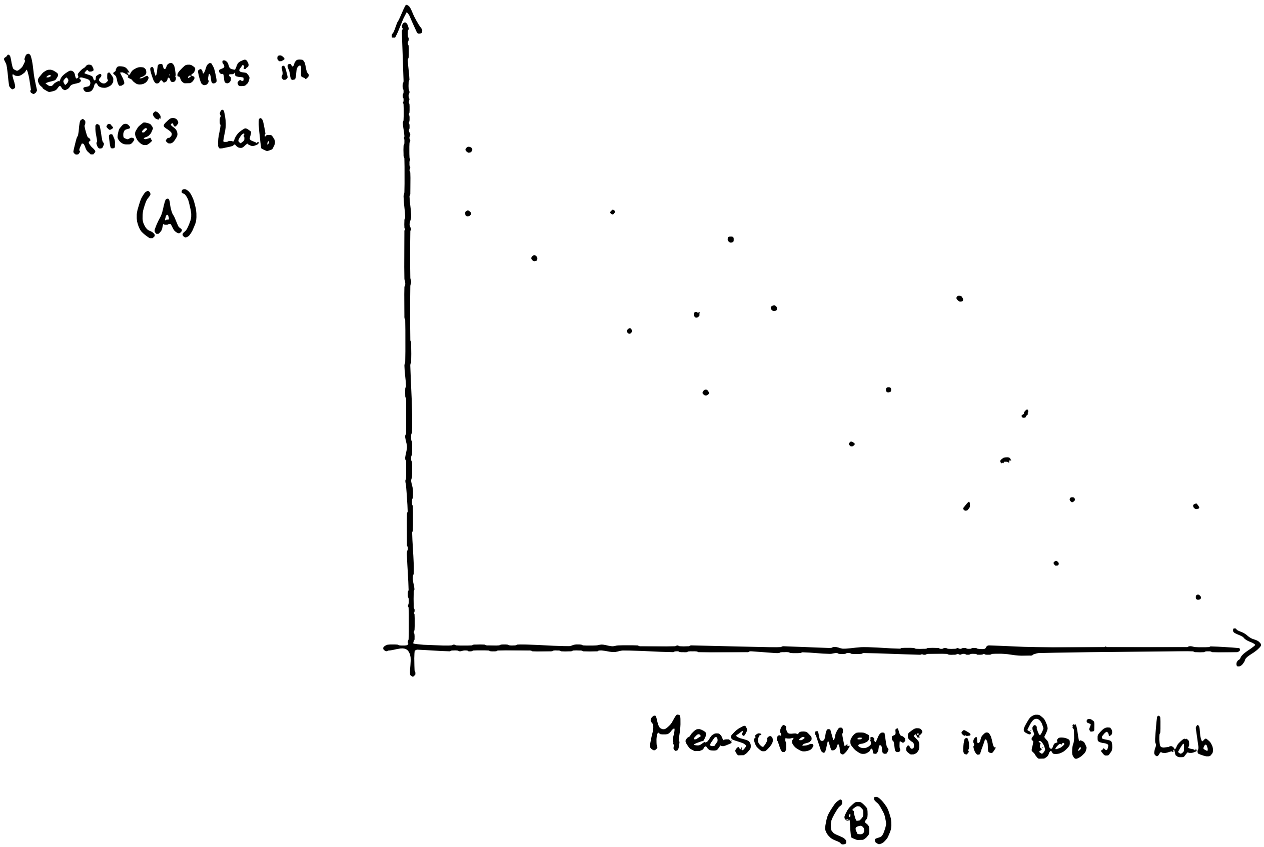
quant-ph/0401052

Operational Quantum Mechanics

$P =$  $\longleftrightarrow \rho_P$

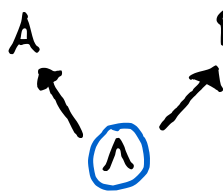
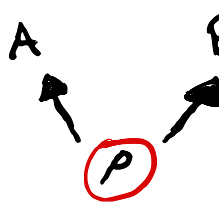
$M =$  $\longleftrightarrow \{E_k^M\}$

$$\boxed{\Pr(k \mid P, M) = \mathrm{Tr}(\rho_P E_k^M)}$$



Measurements in Alice's Lab
(A)
Measurements in Bob's Lab
(B)

Reichenbach's Principle and Quantum Causes

Theory:	Causal Hypothesis:	$P(A,B)$:
Classical		$\sum_{\Lambda} P(A \mid \Lambda)\, P(B \mid \Lambda)\, P(\Lambda)$
Quantum		$\mathrm{Tr}(M_A \otimes M_B \rho)$

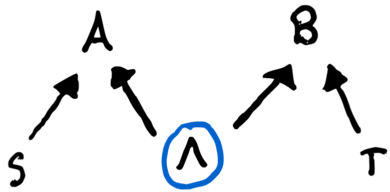
The EPR Paradox

$$\mathcal{D}\left(\begin{tikzcd} A & & B \\ & \Lambda \arrow[ul] \arrow[ur] & \end{tikzcd}\right) = \mathcal{D}\left(\begin{tikzcd} A & & B \\ & \rho \arrow[ul] \arrow[ur] & \end{tikzcd}\right)$$

Not quintessentially quantum!

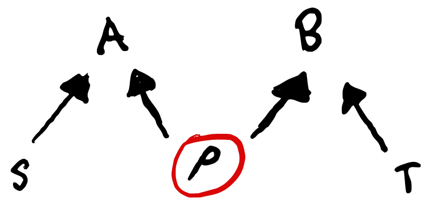
Theory: **Causal Hypothesis:** **$P(A,B \mid S,T)$:**

Classical



$$\sum_{\Lambda} P(A \mid S,\Lambda)\, P(B \mid T,\Lambda)\, P(\Lambda)$$

Quantum



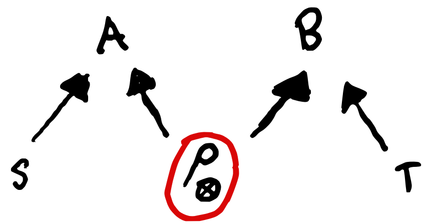
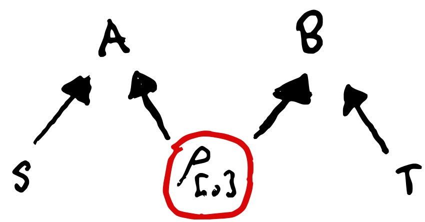
$$\mathrm{Tr}\left(M_{A|S} \otimes M_{B|T}\, \rho\right)$$

Bell's Theorem

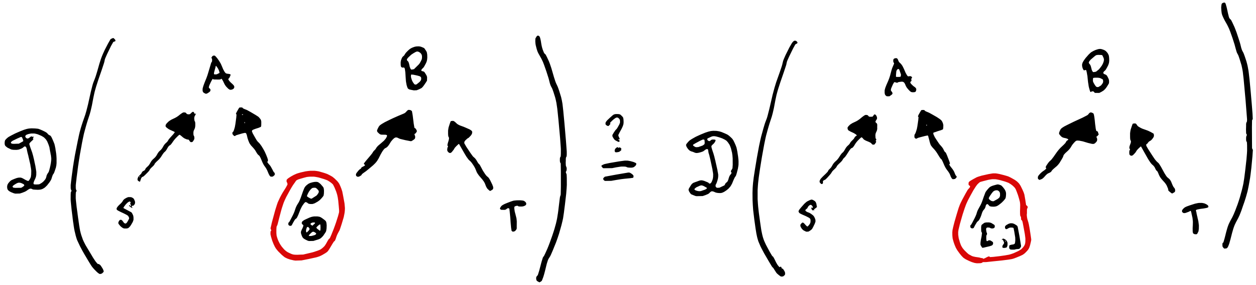
$$\mathcal{D}\left(\begin{tikzcd} & A & & B & \\ S \arrow[ur] & & \Lambda \arrow[ul] \arrow[ur] & & T \arrow[ul] \end{tikzcd}\right) \subsetneq \mathcal{D}\left(\begin{tikzcd} & A & & B & \\ S \arrow[ur] & & \rho \arrow[ul] \arrow[ur] & & T \arrow[ul] \end{tikzcd}\right)$$

Quintessentially quantum!

- Example of a "quantum-classical gap" (QC-gap)

Theory:	Causal Hypothesis:	$P(A,B \mid S,T)$:
Quantum$_{\otimes}$		$\mathrm{Tr}(M_{A\mid S} \otimes M_{B\mid T}\, \rho_{\otimes})$
Quantum$_{[,]}$		$\mathrm{Tr}(M_{A\mid S} M_{B\mid T}\, \rho_{[,]})$

Tsirelson's Problem ($\dim \mathcal{H}_{AB} = \infty$)

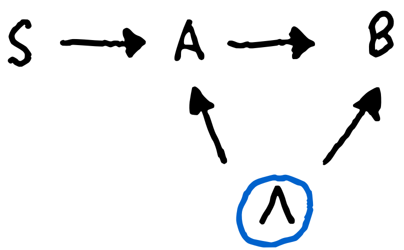


Thm. (Ji et al.) : No!

2001.04383

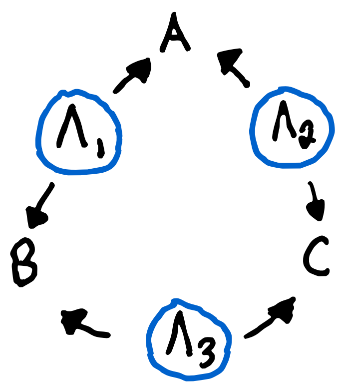
Other Structures with QC-Gaps

Instrumental Scenario:



1804.04119

Triangle Scenario:



1909.10519

Quantum-Classical Gaps (QC-Gaps)

$$G = (V \cup L_c, E)$$

$$QG = (V \cup \tilde{L}_c \cup L_Q, E)$$

Directed Acyclic Graph (DAG)

Def:

$$G \text{ admits QC-gap} \longleftrightarrow \mathcal{D}(G) \subsetneq \mathcal{D}(QG)$$

Thm. (Henson, Lal, Pusey; Evans):

$\exists$ Polynomial $\mathcal{B}$ such that:

$$G \text{ admits QC-gap} \longleftrightarrow \begin{array}{l} \text{(i) } \forall P \in \mathcal{D}(G): \text{“}\mathcal{B}(P) \leq 0\text{”} \\ \text{(ii) } \exists P \in \mathcal{D}(QG): \text{“}\mathcal{B}(P) > 0\text{”} \end{array}$$

(1405.2572, 1408.1809)

Piggyback

"To use something that already exists or has already been done successfully to do something else quickly or effectively."

(Cambridge English Dictionary)

Piggyback Maps

Rough Idea:

Devise QC-gap non-increasing map $\mathfrak{X}$ ("piggyback"):

$$(G, QG) \overset{\mathfrak{X}}{\longmapsto} (G^*, QG^*)$$

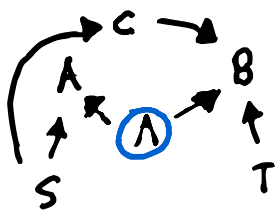
$$\to \quad \mathfrak{D}(QG) \setminus \mathfrak{D}(G) \supseteq \mathfrak{D}(QG^*) \setminus \mathfrak{D}(G^*).$$

Thm:

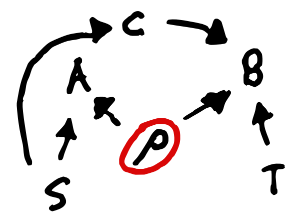
If G^* admits a QC-gap, then G admits a QC-gap.

Point Distribution Piggyback

G

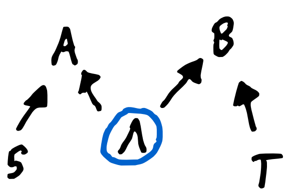


QG

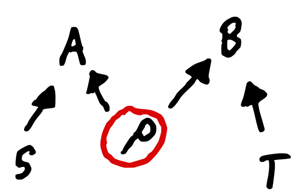


$\mathcal{X}$ = "Fix C to a point distribution"

$G^{\mathcal{X}}$

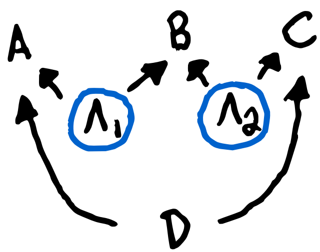


$QG^{\mathcal{X}}$

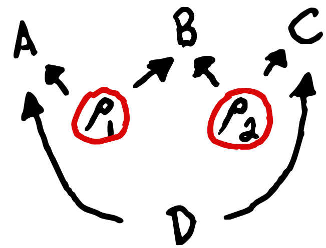


Marginalization Piggyback

G

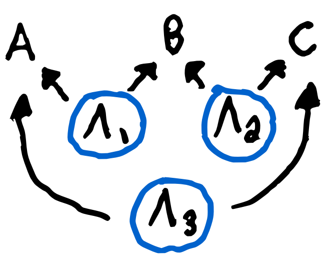


QG

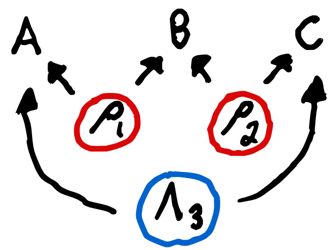


$\mathcal{X}$ = "Marginalize D"

$G^{\mathcal{X}}$

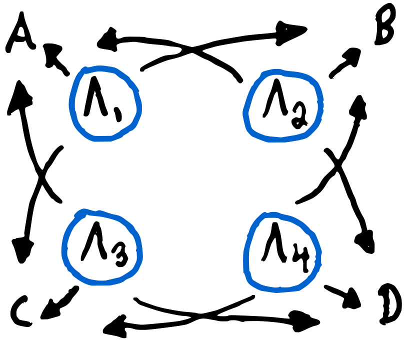


$QG^{\mathcal{X}}$

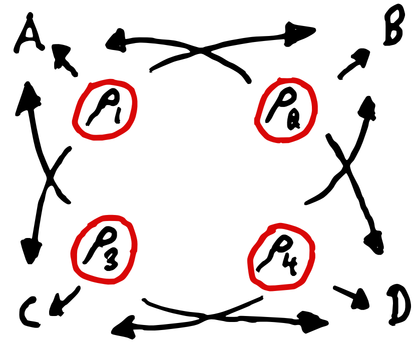


Fritz Piggyback

G

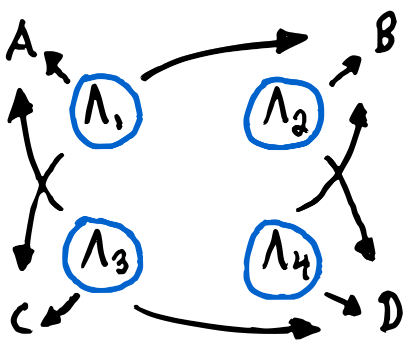


QG

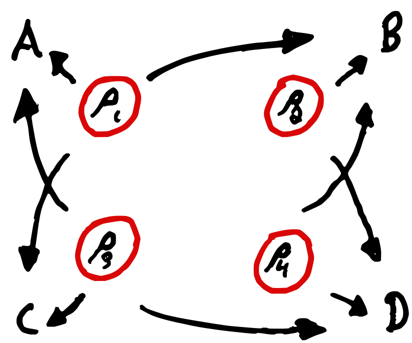


$\mathcal{X}_1$ = "Assume Perfect Correlation between A and C"

$G^{\mathcal{X}_1}$



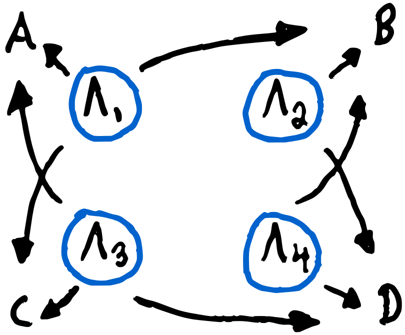
$QG^{\mathcal{X}_1}$



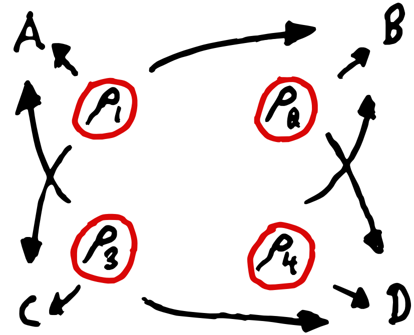
1206.5115

Fritz Piggyback

$G^{\mathcal{X}_1}$

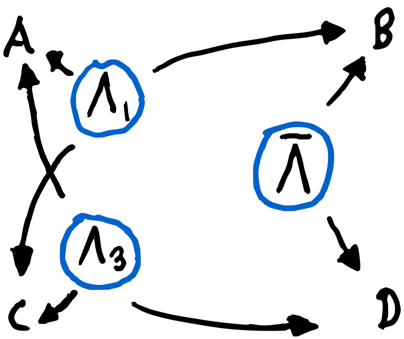


$QG^{\mathcal{X}_1}$

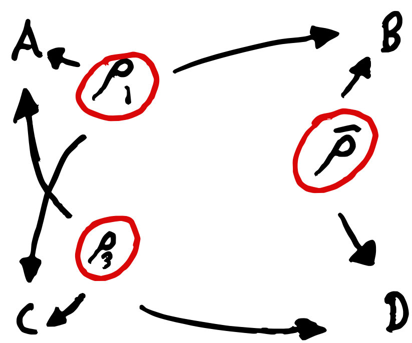


$\mathcal{X}_2 =$ "Consolidate $\Lambda_2, \Lambda_4 / \rho_2, \rho_4$"

$G^{\mathcal{X}_2 \circ \mathcal{X}_1}$

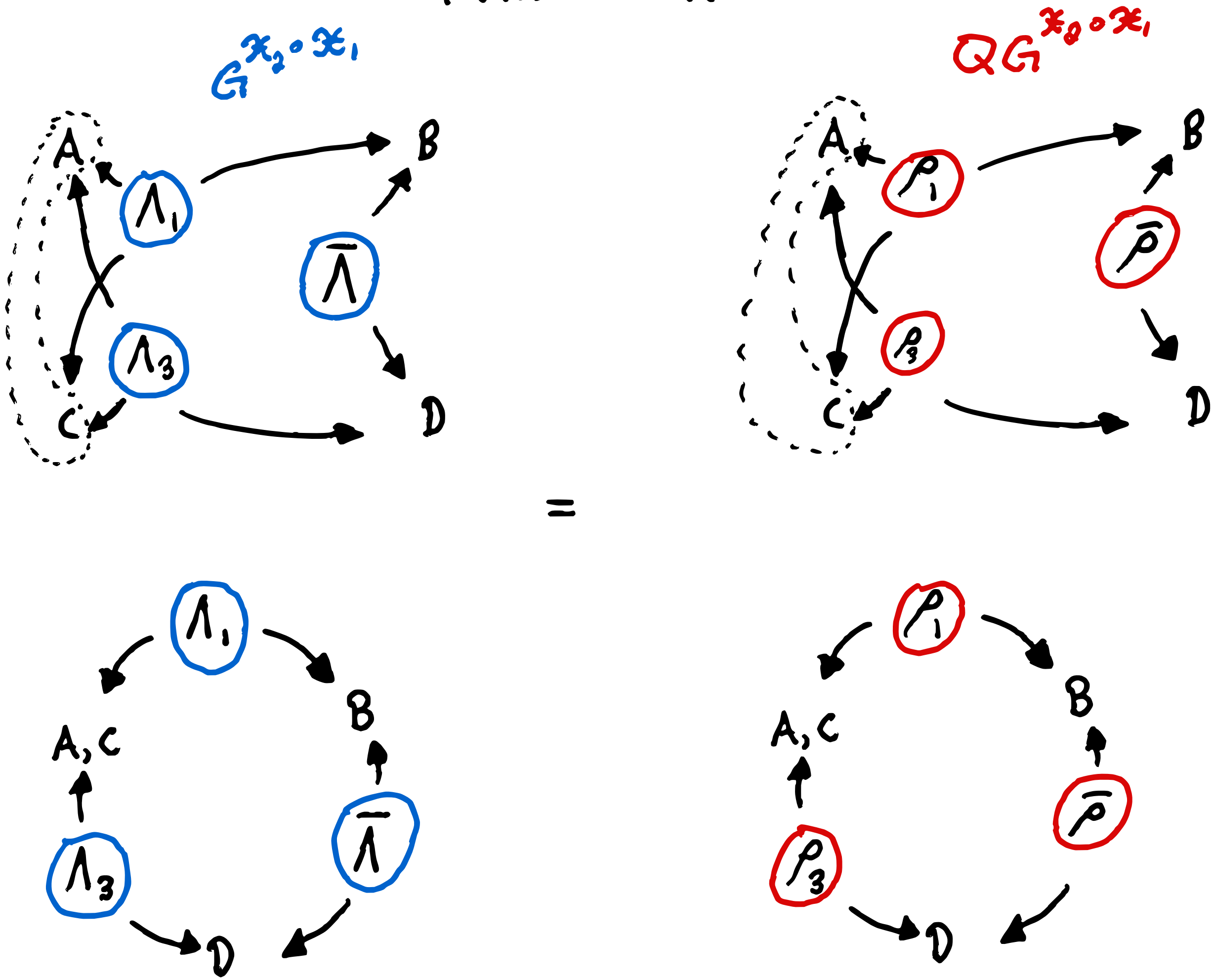


$QG^{\mathcal{X}_2 \circ \mathcal{X}_1}$



1206.5115

Fritz Piggyback



1206.5115

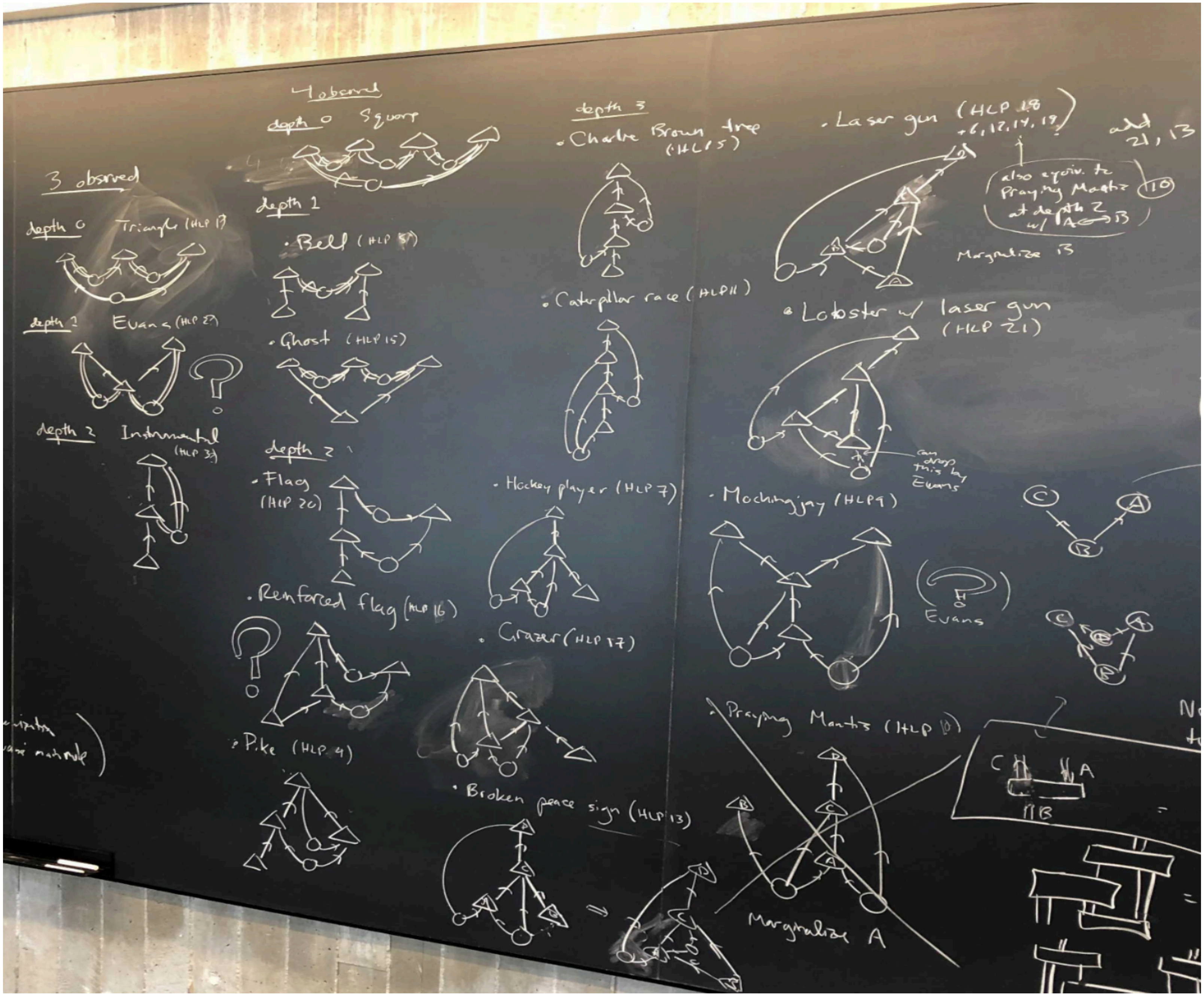
Theory-independent limits on correlations from generalised Bayesian networks

Joe Henson
Imperial College London and
University of Bristol
j.henson@bristol.ac.uk

Raymond Lal
University of Oxford and
University of Cambridge
rayl@cs.ox.ac.uk

Matthew F. Pusey
Perimeter Institute
m@physics.org

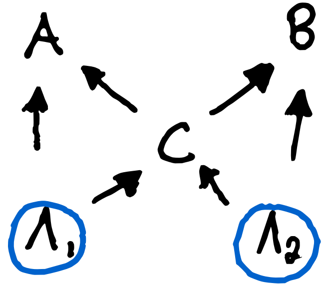
Bayesian networks provide a powerful tool for reasoning about probabilistic causation, used in many areas of science. They are, however, intrinsically classical. In particular, Bayesian networks naturally yield the Bell inequalities. Inspired by this connection, we generalise the formalism of classical Bayesian networks in order to investigate non-classical correlations in arbitrary causal structures. Our framework of 'generalised Bayesian networks' replaces latent variables with the resources of any generalised probabilistic theory, most importantly quantum theory, but also, for example, Popescu-Rohrlich boxes. We obtain three main sets of results. Firstly, we prove that all of the observable conditional independences required by the classical theory also hold in our generalisation; to obtain this, we extend the classical *d*-separation theorem to our setting. Secondly, we find that the theory-independent constraints on probabilities can go beyond these conditional independences. For example we find that no probabilistic theory predicts perfect correlation between three parties using only bipartite common causes. Finally, we begin a classification of those causal structures, such as the Bell scenario, that may yield a separation between classical, quantum and general-probabilistic correlations.



3 observed
depth 0 Triangle (HLP 1)
depth 1 Evans (HLP 2)
depth 2 Instrumental (HLP 3)
4 observed
depth 0 Square
depth 1
• Bell (HLP 5)
• Ghost (HLP 15)
depth 2
• Flag (HLP 20)
• Reinforced flag (HLP 16)
• Pike (HLP 4)
depth 3
• Charlie Brown tree (HLP 15)
• Caterpillar race (HLP 11)
• Hockey player (HLP 7)
• Grazer (HLP 17)
• Broken peace sign (HLP 13)
• Laser gun (HLP 18 +6, 12, 14, 19)
also equiv. to Praying Mantis (10) at depth 2 w/ A ⟷ B
Marginalize B
add 21, 13
• Lobster w/ laser gun (HLP 21)
can drop this by Evans
• Mockingjay (HLP 9)
Evans
• Praying Mantis (HLP 10)
Marginalize A

Some Open Problems

- Does Evan's DAG admit a QC-gap?



(2211.13349)

- If G admits a QC-gap, then does G piggyback Bell?
- If G has inequality constraints, then does G admit a QC-gap?
- Tsirelson's problem in more general causal scenarios.

THANK YOU!

Toward a Formalization of Piggyback Maps

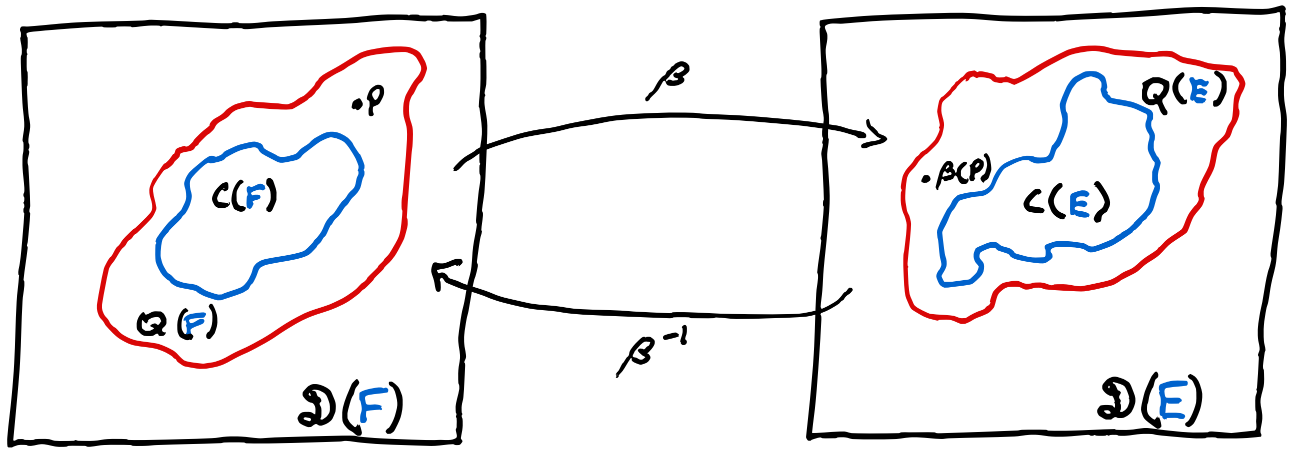
Def: Let F ("familiar") and E ("exotic") be two DAGS, with $S_E \subseteq \mathcal{D}(V_E)$ a set of prob. dist. over the visible vertices of E. A map $\beta : \mathcal{D}(V_F) \longrightarrow S_E$ is a <u>piggyback from F to E</u> iff:

(i) β is invertible,

(ii) $p \in Q(V_F) \Longrightarrow \beta(p) \in Q(V_E)$ ("quantum-preserving from F to E")

(iii) $p' \in C(V_E) \Longrightarrow \beta^{-1}(p') \in C(V_F)$ ("classical-preserving from E to F")

Familiar F Exotic E



<u>Thm.</u> If F admits a QC-gap and $\exists$ piggyback from F to E, then E admits a QC-gap.

The Point Distribution Piggyback

$$P_F(X_1, \dots, X_n) \xrightarrow{\beta} P_E(X_1, \dots, X_n, U = u^*)$$

$$= P_F(X_1, \dots, X_n)$$

(i) β invertible

(ii) $P \in Q(V_F) \Longrightarrow \beta(P) \in Q(V_E)$

(iii) $P' \in C(V_E) \Longrightarrow \beta^{-1}(P') \in C(V_F)$